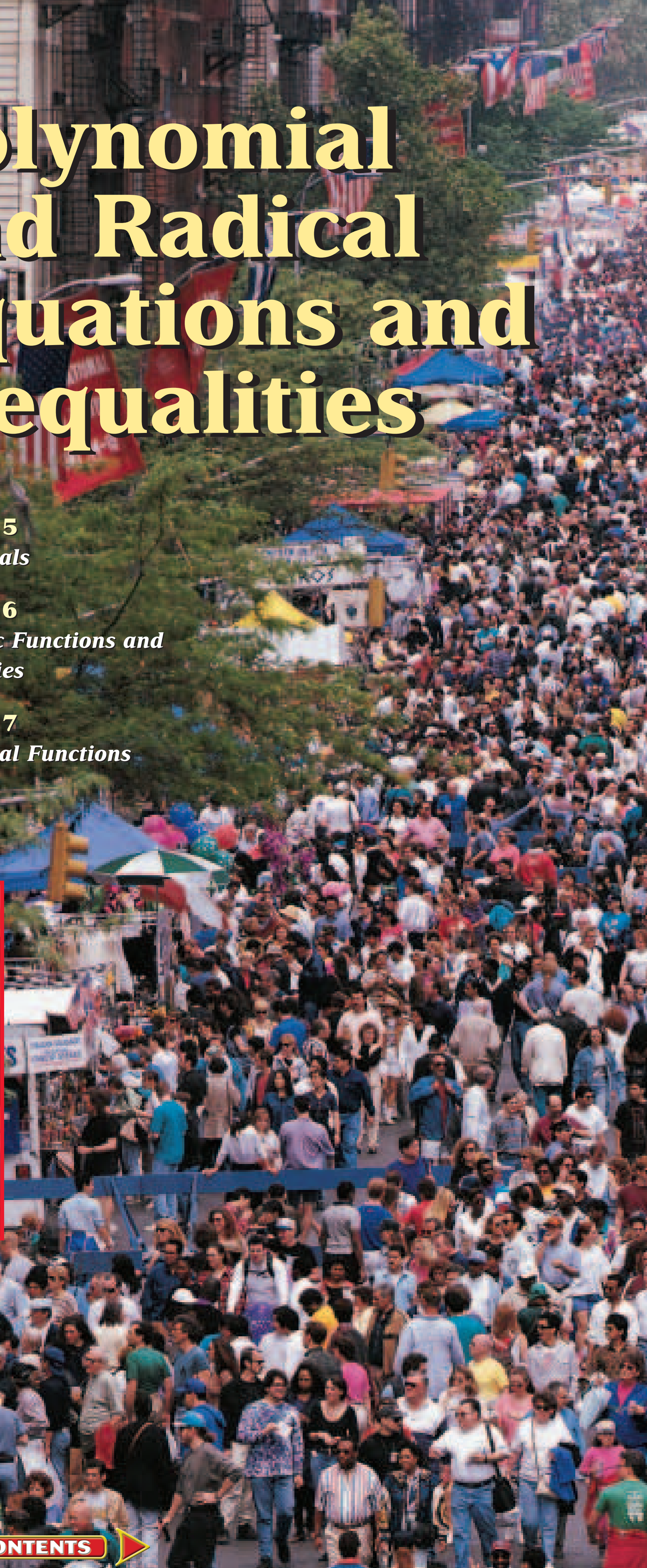
UNIT 2

Polynomial and Radical Equations and Inequalities

Equations that model real-world data allow you to make predictions about the future. In this unit, you will learn about nonlinear equations, including polynomial and radical equations, and inequalities.



CONTENTS

WebQuest Internet Project

Population Explosion

The United Nations estimated that the world's population reached 6 billion in 1999. The population had doubled in about 40 years and gained 1 billion people in just 12 years. Assuming middle-range birth and death trends, world population is expected to exceed 9 billion by 2050, with most of the increase in countries that are less economically developed. In this project, you will use quadratic and polynomial mathematical models that will help you to project future populations.

Log on to www.algebra2.com/webquest.
Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 2.

Lesson	5-1	6-6	7-4
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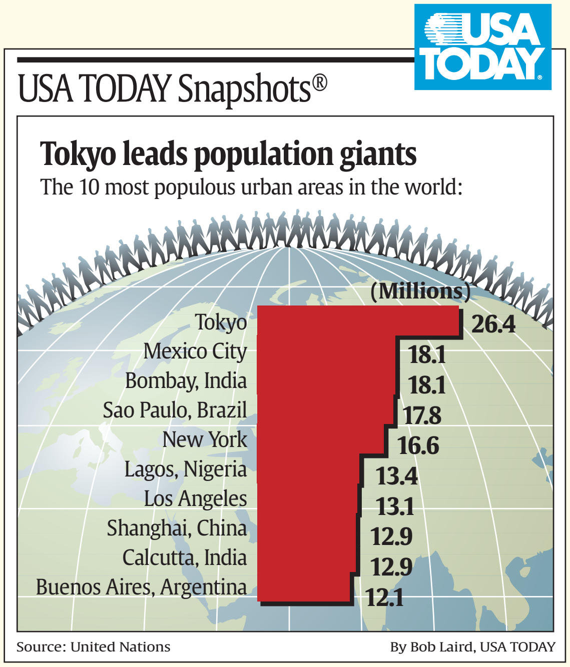


Chapter

5 Polynomials

What You'll Learn

- **Lessons 5-1 through 5-4** Add, subtract, multiply, divide, and factor polynomials.
- **Lessons 5-5 through 5-8** Simplify and solve equations involving roots, radicals, and rational exponents.
- **Lesson 5-9** Perform operations with complex numbers.

Key Vocabulary

- **scientific notation** (p. 225)
- **polynomial** (p. 229)
- **FOIL method** (p. 230)
- **synthetic division** (p. 234)
- **complex number** (p. 271)

Why It's Important

Many formulas involve polynomials and/or square roots. For example, equations involving speeds or velocities of objects are often written with square roots. You can use such an equation to find the velocity of a roller coaster. *You will use an equation relating the velocity of a roller coaster and the height of a hill in Lesson 5-6.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 5.

For Lessons 5-2 and 5-9 — Rewrite Differences as Sums

Rewrite each difference as a sum.

1. $2 - 7$ **2.** $-6 - 11$ **3.** $x - y$

4. $8 - 2x$ **5.** $2xy - 6yz$ **6.** $6a^2b - 12b^2c$

For Lesson 5-2 — Distributive Property

Use the Distributive Property to rewrite each expression without parentheses.
(For review, see Lesson 1-2.)

7. $-2(4x^3 + x - 3)$ **8.** $-1(x + 2)$ **9.** $-1(x - 3)$

10. $-3(2x^4 - 5x^2 - 2)$ **11.** $-\frac{1}{2}(3a + 2)$ **12.** $-\frac{2}{3}(2 + 6z)$

For Lessons 5-5 and 5-9 — Classify Numbers

Find the value of each expression. Then name the sets of numbers to which each value belongs. *(For review, see Lesson 1-2.)*

13. $2.6 + 3.7$ **14.** $18 \div (-3)$ **15.** $2^3 + 3^2$

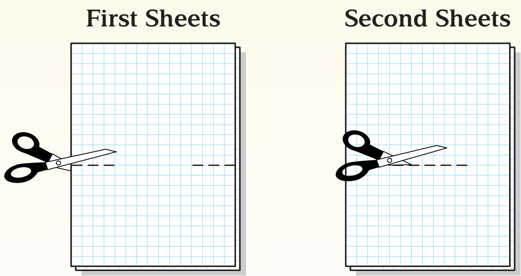
16. $\sqrt{4 + 1}$ **17.** $\frac{18 + 14}{8}$ **18.** $3\sqrt{4}$

FOLDABLES™ Study Organizer

Polynomials Make this Foldable to help you organize your notes. Begin with four sheets of grid paper.

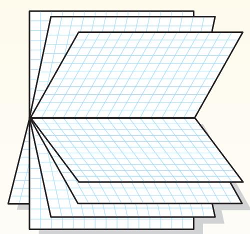
Step 1 Fold and Cut

Fold in half along the width. On the first two sheets, cut along the fold at the ends. On the second two sheets, cut in the center of the fold as shown.



Step 2 Fold and Label

Insert first sheets through second sheets and align the folds. Label the pages with lesson numbers.



Reading and Writing As you read and study the chapter, fill the journal with notes, diagrams, and examples for polynomials.

5-1 Monomials

What You'll Learn

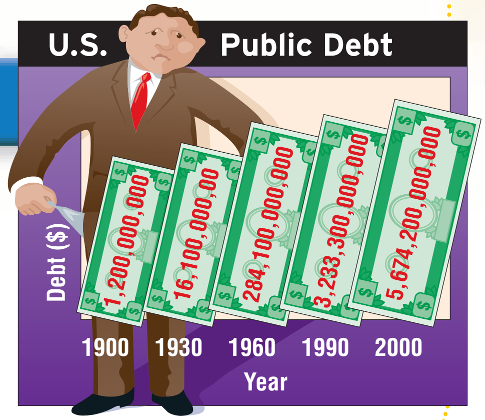
- Multiply and divide monomials.
- Use expressions written in scientific notation.

Vocabulary

- monomial
- constant
- coefficient
- degree
- power
- simplify
- standard notation
- scientific notation
- dimensional analysis

Why is scientific notation useful in economics?

Economists often deal with very large numbers. For example, the table shows the U.S. public debt for several years in the last century. Such numbers, written in standard notation, are difficult to work with because they contain so many digits. Scientific notation uses powers of ten to make very large or very small numbers more manageable.



Source: U.S. Department of the Treasury

MONOMIALS A **monomial** is an expression that is a number, a variable, or the product of a number and one or more variables. Monomials cannot contain variables in denominators, variables with exponents that are negative, or variables under radicals.

Monomials	Not Monomials
$5b, -w, 23, x^2, \frac{1}{3}x^3y^4$	$\frac{1}{n^4}, \sqrt[3]{x}, x + 8, a^{-1}$

Constants are monomials that contain no variables, like 23 or -1. The numerical factor of a monomial is the **coefficient** of the variable(s). For example, the coefficient of m in $-6m$ is -6. The **degree** of a monomial is the sum of the exponents of its variables. For example, the degree of $12g^7h^4$ is $7 + 4$ or 11. The degree of a constant is 0.

A **power** is an expression of the form x^n. The word *power* is also used to refer to the exponent itself. Negative exponents are a way of expressing the multiplicative inverse of a number. For example, $\frac{1}{x^2}$ can be written as x^{-2}. Note that an expression such as x^{-2} is not a monomial. *Why?*

Key Concept — Negative Exponents

- **Words** For any real number $a \neq 0$ and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.
- **Examples** $2^{-3} = \frac{1}{2^3}$ and $\frac{1}{b^{-8}} = b^8$

To **simplify** an expression containing powers means to rewrite the expression without parentheses or negative exponents.

Example 1 Simplify Expressions with Multiplication

Simplify $(3x^3y^2)(-4x^2y^4)$.

$(3x^3y^2)(-4x^2y^4) = (3 \cdot x \cdot x \cdot x \cdot y \cdot y)(-4 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y)$	Definition of exponents
$= 3(-4) \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$	Commutative Property
$= -12x^5y^6$	Definition of exponents

Example 1 suggests the following property of exponents.

Key Concept — ***Product of Powers***

- **Words** For any real number a and integers m and n, $a^m \cdot a^n = a^{m+n}$.
- **Examples** $4^2 \cdot 4^9 = 4^{11}$ and $b^3 \cdot b^5 = b^8$

To multiply powers of the same variable, add the exponents. Knowing this, it seems reasonable to expect that when dividing powers, you would subtract exponents. Consider $\frac{x^9}{x^5}$.

$$\frac{x^9}{x^5} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot x \cdot x \cdot x \cdot x}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}}$$ Remember that $x \neq 0$.

$$= x \cdot x \cdot x \cdot x$$ Simplify.

$$= x^4$$ Definition of exponents

It appears that our conjecture is true. To divide powers of the same base, you subtract exponents.

Key Concept — ***Quotient of Powers***

- **Words** For any real number $a \neq 0$, and integers m and n, $\frac{a^m}{a^n} = a^{m-n}$.
- **Examples** $\frac{5^3}{5} = 5^{3-1}$ or 5^2 and $\frac{x^7}{x^3} = x^{7-3}$ or x^4

Example 2 *Simplify Expressions with Division*

Simplify $\frac{p^3}{p^8}$. Assume that $p \neq 0$.

$$\frac{p^3}{p^8} = p^{3-8}$$ Subtract exponents.

$$= p^{-5} \text{ or } \frac{1}{p^5}$$ Remember that a simplified expression cannot contain negative exponents.

CHECK $$\frac{p^3}{p^8} = \frac{\overset{1}{\cancel{p}} \cdot \overset{1}{\cancel{p}} \cdot \overset{1}{\cancel{p}}}{\underset{1}{\cancel{p}} \cdot \underset{1}{\cancel{p}} \cdot \underset{1}{\cancel{p}} \cdot p \cdot p \cdot p \cdot p \cdot p}$$ Definition of exponents

$$= \frac{1}{p^5}$$ Simplify.

You can use the Quotient of Powers property and the definition of exponents to simplify $\frac{y^4}{y^4}$, if $y \neq 0$.

Method 1

$$\frac{y^4}{y^4} = y^{4-4}$$ Quotient of Powers

$$= y^0$$ Subtract.

Method 2

$$\frac{y^4}{y^4} = \frac{\overset{1}{\cancel{y}} \cdot \overset{1}{\cancel{y}} \cdot \overset{1}{\cancel{y}} \cdot \overset{1}{\cancel{y}}}{\underset{1}{\cancel{y}} \cdot \underset{1}{\cancel{y}} \cdot \underset{1}{\cancel{y}} \cdot \underset{1}{\cancel{y}}}$$ Definition of exponents

$$= 1$$ Divide.

In order to make the results of these two methods consistent, we define $y^0 = 1$, where $y \neq 0$. In other words, any nonzero number raised to the zero power is equal to 1. *Notice that 0^0 is undefined.*

www.algebra2.com/extra_examples

The properties we have presented can be used to verify the properties of powers that are listed below.

Key Concept — *Properties of Powers*

- **Words** Suppose a and b are real numbers and m and n are integers. Then the following properties hold.

Words	Examples
Power of a Power: $(a^m)^n = a^{mn}$	$(a^2)^3 = a^6$
Power of a Product: $(ab)^m = a^m b^m$	$(xy)^2 = x^2y^2$
Power of a Quotient: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$ and	$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ or $\frac{b^n}{a^n}$, $a \neq 0$, $b \neq 0$	$\left(\frac{x}{y}\right)^{-4} = \frac{y^4}{x^4}$

Example 3 Simplify Expressions with Powers

Simplify each expression.

a. $(a^3)^6$

$(a^3)^6 = a^{3(6)}$ Power of a power

$= a^{18}$

b. $(-2p^3s^2)^5$

$(-2p^3s^2)^5 = (-2)^5 \cdot (p^3)^5 \cdot (s^2)^5$

$= -32p^{15}s^{10}$ Power of a power

c. $\left(\frac{-3x}{y}\right)^4$

$\left(\frac{-3x}{y}\right)^4 = \frac{(-3x)^4}{y^4}$ Power of a quotient

$= \frac{(-3)^4x^4}{y^4}$ Power of a product

$= \frac{81x^4}{y^4}$ $(-3)^4 = 81$

d. $\left(\frac{a}{4}\right)^{-3}$

$\left(\frac{a}{4}\right)^{-3} = \left(\frac{4}{a}\right)^3$ Negative exponent

$= \frac{4^3}{a^3}$ Power of a quotient

$= \frac{64}{a^3}$ $4^3 = 64$

Study Tip

Simplified Expressions

A monomial expression is in simplified form when:

- there are no powers of powers,
- each base appears exactly once,
- all fractions are in simplest form, and
- there are no negative exponents.

With complicated expressions, you often have a choice of which way to start simplifying.

Example 4 Simplify Expressions Using Several Properties

Simplify $\left(\frac{-2x^{3n}}{x^{2n}y^3}\right)^4$.

Method 1

Raise the numerator and denominator to the fourth power before simplifying.

$$\left(\frac{-2x^{3n}}{x^{2n}y^3}\right)^4 = \frac{(-2x^{3n})^4}{(x^{2n}y^3)^4}$$

$$= \frac{(-2)^4(x^{3n})^4}{(x^{2n})^4(y^3)^4}$$

$$= \frac{16x^{12n}}{x^{8n}y^{12}}$$

$$= \frac{16x^{12n-8n}}{y^{12}}$$

$$= \frac{16x^{4n}}{y^{12}}$$

Method 2

Simplify the fraction before raising to the fourth power.

$$\left(\frac{-2x^{3n}}{x^{2n}y^3}\right)^4 = \left(\frac{-2x^{3n-2n}}{y^3}\right)^4$$

$$= \left(\frac{-2x^n}{y^3}\right)^4$$

$$= \frac{16x^{4n}}{y^{12}}$$

SCIENTIFIC NOTATION The form that you usually write numbers in is **standard notation**. A number is in **scientific notation** when it is in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer. Scientific notation is used to express very large or very small numbers.

Study Tip

Graphing Calculators

To solve scientific notation problems on a graphing calculator, use the EE function. Enter 6.38×10^6 as 6.38 [2nd] [EE] 6.

Example 5 Express Numbers in Scientific Notation

Express each number in scientific notation.

a. 6,380,000

$6{,}380{,}000 = 6.38 \times 1{,}000{,}000$ — $1 \leq 6.38 < 10$

$= 6.38 \times 10^6$ — Write 1,000,000 as a power of 10.

b. 0.000047

$0.000047 = 4.7 \times 0.00001$ — $1 \leq 4.7 < 10$

$= 4.7 \times \frac{1}{10^5}$ — $0.00001 = \frac{1}{100{,}000}$ or $\frac{1}{10^5}$

$= 4.7 \times 10^{-5}$ — Use a negative exponent.

You can use properties of powers to multiply and divide numbers in scientific notation.

Example 6 Multiply Numbers in Scientific Notation

Evaluate. Express the result in scientific notation.

a. $(4 \times 10^5)(2 \times 10^7)$

$(4 \times 10^5)(2 \times 10^7) = (4 \cdot 2) \times (10^5 \cdot 10^7)$ — Associative and Commutative Properties

$= 8 \times 10^{12}$ — $4 \cdot 2 = 8$, $10^5 \cdot 10^7 = 10^{5+7}$ or 10^{12}

b. $(2.7 \times 10^{-2})(3 \times 10^6)$

$(2.7 \times 10^{-2})(3 \times 10^6) = (2.7 \cdot 3) \times (10^{-2} \cdot 10^6)$ — Associative and Commutative Properties

$= 8.1 \times 10^4$ — $2.7 \cdot 3 = 8.1$, $10^{-2} \cdot 10^6 = 10^{-2+6}$ or 10^4

Real-world problems often involve units of measure. Performing operations with units is known as **dimensional analysis**.

More About...

Astronomy

Light travels at a speed of about 3.00×10^8 m/s. The distance that light travels in a year is called a *light-year*.

Source: www.britannica.com

Example 7 Divide Numbers in Scientific Notation

ASTRONOMY After the Sun, the next-closest star to Earth is Alpha Centauri C, which is about 4×10^{16} meters away. How long does it take light from Alpha Centauri C to reach Earth? Use the information at the left.

Begin with the formula $d = rt$, where d is distance, r is rate, and t is time.

$t = \frac{d}{r}$ — Solve the formula for time.

$= \frac{4 \times 10^{16} \text{ m}}{3.00 \times 10^8 \text{ m/s}}$ — ← Distance from Alpha Centauri C to Earth; ← speed of light

$= \frac{4}{3.00} \cdot \frac{10^{16}}{10^8 \text{ 1/s}}$ — **Estimate:** The result should be slightly greater than $\frac{10^{16}}{10^8}$ or 10^8.

$\approx 1.33 \times 10^8$ s — $\frac{4}{3.00} \approx 1.33$, $\frac{10^{16}}{10^8} = 10^{16-8}$ or 10^8

It takes about 1.33×10^8 seconds or 4.2 years for light from Alpha Centauri C to reach Earth.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write an example that illustrates a property of powers. Then use multiplication or division to explain why it is true.

2. **Determine** whether $x^y \cdot x^z = x^{yz}$ is *sometimes*, *always*, or *never* true. Explain.

3. **FIND THE ERROR** Alejandra and Kyle both simplified $\frac{2a^2b}{(-2ab^3)^{-2}}$.

Alejandra

$$\frac{2a^2b}{(-2ab^3)^{-2}} = (2a^2b)(-2ab^3)^2$$
$$= (2a^2b)(-2)^2a^2(b^3)^2$$
$$= (2a^2b)2^2a^2b^6$$
$$= 8a^4b^7$$

Kyle

$$\frac{2a^2b}{(-2ab^3)^{-2}} = \frac{2a^2b}{(-2)^{-2}a(b^3)^{-2}}$$
$$= \frac{2a^2b}{4ab^{-6}}$$
$$= \frac{2a^2bb^6}{4a}$$
$$= \frac{ab^7}{2}$$

Who is correct? Explain your reasoning.

Guided Practice

Simplify. Assume that no variable equals 0.

4. $x^2 \cdot x^8$
5. $(2b)^4$
6. $(n^3)^3(n^{-3})^3$
7. $\frac{30y^4}{-5y^2}$
8. $\frac{-2a^3b^6}{18a^2b^2}$
9. $\frac{81p^6q^5}{(3p^2q)^2}$
10. $\left(\frac{1}{w^4z^2}\right)^3$
11. $\left(\frac{cd}{3}\right)^{-2}$
12. $\left(\frac{-6x^6}{3x^3}\right)^{-2}$

Express each number in scientific notation.

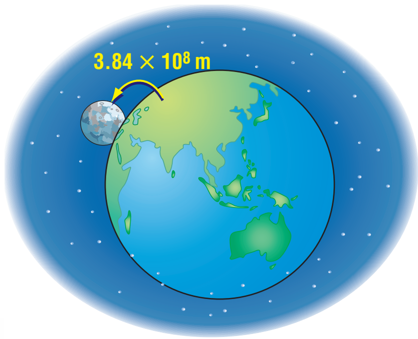
13. 421,000
14. 0.000862

Evaluate. Express the result in scientific notation.

15. $(3.42 \times 10^8)(1.1 \times 10^{-5})$
16. $\frac{8 \times 10^{-1}}{16 \times 10^{-2}}$

Application

17. **ASTRONOMY** Refer to Example 7 on page 225. The average distance from Earth to the Moon is about 3.84×10^8 meters. How long would it take a radio signal traveling at the speed of light to cover that distance?



Practice and Apply

Simplify. Assume that no variable equals 0.

18. $a^2 \cdot a^6$
19. $b^{-3} \cdot b^7$
20. $(n^4)^4$
21. $(z^2)^5$
22. $(2x)^4$
23. $(-2c)^3$
24. $\frac{a^2n^6}{an^5}$
25. $\frac{-y^5z^7}{y^2z^5}$
26. $(7x^3y^{-5})(4xy^3)$
27. $(-3b^3c)(7b^2c^2)$
28. $(a^3b^3)(ab)^{-2}$
29. $(-2r^2s)^3(3rs^2)$
30. $2x^2(6y^3)(2x^2y)$
31. $3a(5a^2b)(6ab^3)$
32. $\frac{-5x^3y^3z^4}{20x^3y^7z^4}$

Homework Help

For Exercises	See Examples
18–35, 60	1–3
36–39	4
40–43	1, 2
44–49, 56, 57	5
50–55, 58, 59	6, 7

Extra Practice
See page 836.

33. $\dfrac{3a^5b^3c^3}{9a^3b^7c}$ **34.** $\dfrac{2c^3d(3c^2d^5)}{30c^4d^2}$ **35.** $\dfrac{-12m^4n^8(m^3n^2)}{36m^3n}$

36. $\left(\dfrac{8a^3b^2}{16a^2b^3}\right)^4$ **37.** $\left(\dfrac{6x^2y^4}{3x^4y^3}\right)^3$ **38.** $\left(\dfrac{x}{y^{-1}}\right)^{-2}$

39. $\left(\dfrac{v}{w^{-2}}\right)^{-3}$ **40.** $\dfrac{30a^{-2}b^{-6}}{60a^{-6}b^{-8}}$ **41.** $\dfrac{12x^{-3}y^{-2}z^{-8}}{30x^{-6}y^{-4}z^{-1}}$

42. If $2^{r+5} = 2^{2r-1}$, what is the value of r?

43. What value of r makes $y^{28} = y^{3r} \cdot y^7$ true?

Express each number in scientific notation.

44. 462.3 **45.** 43,200 **46.** 0.0001843

47. 0.006810 **48.** 502,020,000 **49.** 675,400,000

Evaluate. Express the result in scientific notation.

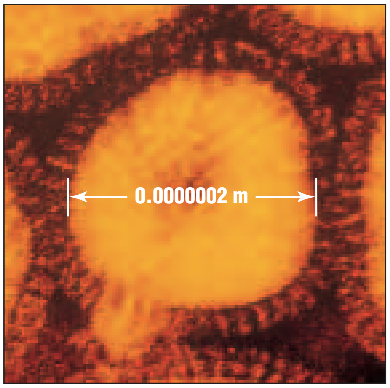
50. $(4.15 \times 10^3)(3.0 \times 10^6)$ **51.** $(3.01 \times 10^{-2})(2 \times 10^{-3})$

52. $\dfrac{6.3 \times 10^5}{1.4 \times 10^3}$ **53.** $\dfrac{9.3 \times 10^7}{1.5 \times 10^{-3}}$

54. $(6.5 \times 10^4)^2$ **55.** $(4.1 \times 10^{-4})^2$

56. POPULATION The population of Earth is about 6,080,000,000. Write this number in scientific notation.

57. BIOLOGY Use the diagram at the right to write the diameter of a typical flu virus in scientific notation.



58. CHEMISTRY One gram of water contains about 3.34×10^{22} molecules. About how many molecules are in 500 grams of water?

59. RESEARCH Use the Internet or other source to find the masses of Earth and the Sun. About how many times as large as Earth is the Sun?

60. CRITICAL THINKING Determine which is greater, 100^{10} or 10^{100}. Explain.

CRITICAL THINKING **For Exercises 61 and 62, use the following proof of the Power of a Power Property.**

$$a^m a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ factors}} \cdot \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}$$
$$= \underbrace{a \cdot a \cdot \ldots \cdot a}_{m + n \text{ factors}}$$
$$= a^{m+n}$$

61. What definition or property allows you to make each step of the proof?

62. Prove the Power of a Product Property, $(ab)^m = a^m b^m$.

63. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

Why is scientific notation useful in economics?

Include the following in your answer:

- the 2000 national debt of \$5,674,200,000,000 and the U.S. population of 281,000,000, both written in words and in scientific notation, and
- an explanation of how to find the amount of debt per person, with the result written in scientific notation and in standard notation.

WebQuest

A scatter plot of populations will help you make a model for the data. Visit www.algebra2.com/webquest to continue work on your WebQuest project.

Standardized Test Practice

64. Simplify $\frac{(2x^2)^3}{12x^4}$.

(A) $\frac{x}{2}$ (B) $\frac{2x}{3}$ (C) $\frac{1}{2x^2}$ (D) $\frac{2x^2}{3}$

65. $7.3 \times 10^5 = ?$

(A) 73,000 (B) 730,000 (C) 7,300,000 (D) 73,000,000

Maintain Your Skills

Mixed Review

Solve each system of equations by using inverse matrices. *(Lesson 4-8)*

66. $2x + 3y = 8$
$x - 2y = -3$

67. $x + 4y = 9$
$3x + 2y = -3$

Find the inverse of each matrix, if it exists. *(Lesson 4-7)*

68. $\begin{bmatrix} 2 & 5 \\ -1 & -2 \end{bmatrix}$

69. $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

Evaluate each determinant. *(Lesson 4-3)*

70. $\begin{vmatrix} 3 & 0 \\ 2 & -2 \end{vmatrix}$

71. $\begin{vmatrix} 1 & 0 & -3 \\ 2 & -1 & 4 \\ -3 & 0 & 2 \end{vmatrix}$

Solve each system of equations. *(Lesson 3-5)*

72. $x + y = 5$
$x + y + z = 4$
$2x - y + 2z = -1$

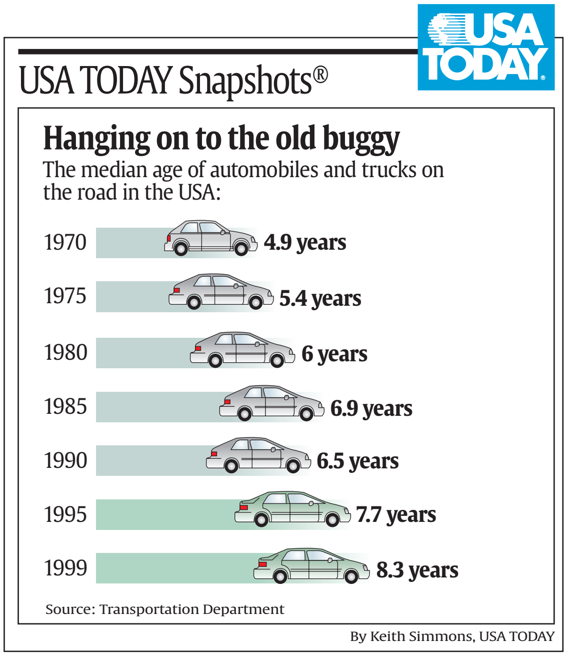
73. $a + b + c = 6$
$2a - b + 3c = 16$
$a + 3b - 2c = -6$

TRANSPORTATION **For Exercises 74–76, refer to the graph at the right.** *(Lesson 2-5)*

74. Make a scatter plot of the data, where the horizontal axis is the number of years since 1970.

75. Write a prediction equation.

76. Predict the median age of vehicles on the road in 2010.



Solve each equation. *(Lesson 1-3)*

77. $2x + 11 = 25$

78. $-12 - 5x = 3$

Getting Ready for the Next Lesson

Use the Distributive Property to find each product.
(To review the ***Distributive Property****, see Lesson 1-2.)*

79. $2(x + y)$ **80.** $3(x - z)$ **81.** $4(x + 2)$

82. $-2(3x - 5)$ **83.** $-5(x - 2y)$ **84.** $-3(-y + 5)$

5-2 Polynomials

What You'll Learn

- Add and subtract polynomials.
- Multiply polynomials.

How can polynomials be applied to financial situations?

Year	Tuition
1	\$8820
2	\$9173
3	\$9540
4	\$9921

Shenequa wants to attend Purdue University in Indiana, where the out-of-state tuition is \$8820. Suppose the tuition increases at a rate of 4% per year. You can use polynomials to represent the increasing tuition costs.

Vocabulary

- polynomial
- terms
- like terms
- trinomial
- binomial
- FOIL method

ADD AND SUBTRACT POLYNOMIALS If r represents the rate of increase of tuition, then the tuition for the second year will be $8820(1 + r)$. For the third year, it will be $8820(1 + r)^2$, or $8820r^2 + 17{,}640r + 8820$ in expanded form. This expression is called a polynomial. A **polynomial** is a monomial or a sum of monomials.

The monomials that make up a polynomial are called the **terms** of the polynomial. In a polynomial such as $x^2 + 2x + x + 1$, the two monomials $2x$ and x can be combined because they are **like terms**. The result is $x^2 + 3x + 1$. The polynomial $x^2 + 3x + 1$ is a **trinomial** because it has three unlike terms. A polynomial such as $xy + z^3$ is a **binomial** because it has two unlike terms. The *degree* of a polynomial is the degree of the monomial with the greatest degree. For example, the degree of $x^2 + 3x + 1$ is 2, and the degree of $xy + z^3$ is 3.

Study Tip

Reading Math

The prefix *bi-* means *two*, and the prefix *tri-* means *three*.

Example 1 Degree of a Polynomial

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

a. $\frac{1}{6}x^3y^5 - 9x^4$

This expression is a polynomial because each term is a monomial.

The degree of the first term is 3 + 5 or 8, and the degree of the second term is 4. The degree of the polynomial is 8.

b. $x + \sqrt{x} + 5$

This expression is not a polynomial because $\sqrt{x}$ is not a monomial.

To *simplify* a polynomial means to perform the operations indicated and combine like terms.

Example 2 Subtract and Simplify

Simplify $(3x^2 - 2x + 3) - (x^2 + 4x - 2)$.

$(3x^2 - 2x + 3) - (x^2 + 4x - 2) = 3x^2 - 2x + 3 - x^2 - 4x + 2$ Distribute the −1.

$= (3x^2 - x^2) + (-2x - 4x) + (3 + 2)$ Group like terms.

$= 2x^2 - 6x + 5$ Combine like terms.

www.algebra2.com/extra_examples

CONTENTS

MULTIPLY POLYNOMIALS You can use the Distributive Property to multiply polynomials.

Example 3 Multiply and Simplify

Find $2x(7x^2 - 3x + 5)$.

$2x(7x^2 - 3x + 5) = 2x(7x^2) + 2x(-3x) + 2x(5)$ Distributive Property

$= 14x^3 - 6x^2 + 10x$ Multiply the monomials.

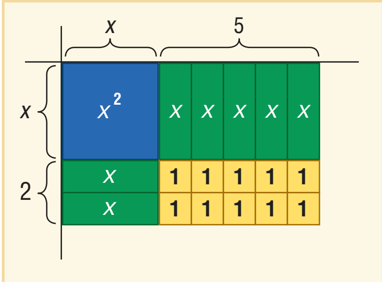
You can use algebra tiles to model the product of two binomials.

Algebra Activity

Multiplying Binomials

Use algebra tiles to find the product of $x + 5$ and $x + 2$.

- Draw a 90° angle on your paper.
- Use an x tile and a 1 tile to mark off a length equal to $x + 5$ along the top.
- Use the tiles to mark off a length equal to $x + 2$ along the side.
- Draw lines to show the grid formed.
- Fill in the lines with the appropriate tiles to show the area product. The model shows the polynomial $x^2 + 7x + 10$.



The area of the rectangle is the product of its length and width. So, $(x + 5)(x + 2) = x^2 + 7x + 10$.

The **FOIL method** uses the Distributive Property to multiply binomials.

Key Concept — FOIL Method for Multiplying Binomials

The product of two binomials is the sum of the products of **F** the *first* terms, **O** the *outer* terms, **I** the *inner* terms, and **L** the *last* terms.

Study Tip

Vertical Method

You may also want to use the vertical method to multiply polynomials.

$$\begin{array}{r} 3y + 2 \\ (\times)\ 5y + 4 \\ \hline 12y + 8 \\ 15y^2 + 10y \\ \hline 15y^2 + 22y + 8 \end{array}$$

Example 4 Multiply Two Binomials

Find $(3y + 2)(5y + 4)$.

$(3y + 2)(5y + 4) = \underbrace{3y \cdot 5y}_{\text{First terms}} + \underbrace{3y \cdot 4}_{\text{Outer terms}} + \underbrace{2 \cdot 5y}_{\text{Inner terms}} + \underbrace{2 \cdot 4}_{\text{Last terms}}$

$= 15y^2 + 22y + 8$ Multiply monomials and add like terms.

Example 5 Multiply Polynomials

Find $(n^2 + 6n - 2)(n + 4)$.

$(n^2 + 6n - 2)(n + 4)$

$= n^2(n + 4) + 6n(n + 4) + (-2)(n + 4)$ Distributive Property

$= n^2 \cdot n + n^2 \cdot 4 + 6n \cdot n + 6n \cdot 4 + (-2) \cdot n + (-2) \cdot 4$ Distributive Property

$= n^3 + 4n^2 + 6n^2 + 24n - 2n - 8$ Multiply monomials.

$= n^3 + 10n^2 + 22n - 8$ Combine like terms.

CONTENTS

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a polynomial of degree 5 that has three terms.
2. **Identify** the degree of the polynomial $2x^3 - x^2 + 3x^4 - 7$.
3. **Model** $3x(x + 2)$ using algebra tiles.

Guided Practice

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

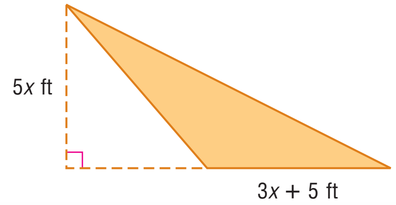
4. $2a + 5b$
5. $\frac{1}{3}x^3 - 9y$
6. $\frac{mw^2 - 3}{nz^3 + 1}$

Simplify.

7. $(2a + 3b) + (8a - 5b)$
8. $(x^2 - 4x + 3) - (4x^2 + 3x - 5)$
9. $2x(3y + 9)$
10. $2p^2q(5pq - 3p^3q^2 + 4pq^4)$
11. $(y - 10)(y + 7)$
12. $(x + 6)(x + 3)$
13. $(2z - 1)(2z + 1)$
14. $(2m - 3n)^2$

Application

15. **GEOMETRY** Find the area of the triangle.



Practice and Apply

Homework Help

For Exercises	See Examples
16–21	1
22–27, 35, 36, 51	2
28–33, 47, 48	3
34	2, 3
37–46, 52, 53	4
49, 50, 54	5

Extra Practice
See page 837.

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

16. $3z^2 - 5z + 11$
17. $x^3 - 9$
18. $\frac{6xy}{z} - \frac{3c}{d}$
19. $\sqrt{m - 5}$
20. $5x^2y^4 + x\sqrt{3}$
21. $\frac{4}{3}y^2 + \frac{5}{6}y^7$

Simplify.

22. $(3x^2 - x + 2) + (x^2 + 4x - 9)$
23. $(5y + 3y^2) + (-8y - 6y^2)$
24. $(9r^2 + 6r + 16) - (8r^2 + 7r + 10)$
25. $(7m^2 + 5m - 9) + (3m^2 - 6)$
26. $(4x^2 - 3y^2 + 5xy) - (8xy + 3y^2)$
27. $(10x^2 - 3xy + 4y^2) - (3x^2 + 5xy)$
28. $4b(cb - zd)$
29. $4a(3a^2 + b)$
30. $-5ab^2(-3a^2b + 6a^3b - 3a^4b^4)$
31. $2xy(3xy^3 - 4xy + 2y^4)$
32. $\frac{3}{4}x^2(8x + 12y - 16xy^2)$
33. $\frac{1}{2}a^3(4a - 6b + 8ab^4)$

34. **PERSONAL FINANCE** Toshiro has \$850 to invest. He can invest in a savings account that has an annual interest rate of 3.7%, and he can invest in a money market account that pays about 5.5% per year. Write a polynomial to represent the amount of interest he will earn in 1 year if he invests x dollars in the savings account and the rest in the money market account.

E-SALES **For Exercises 35 and 36, use the following information.**
A small online retailer estimates that the cost, in dollars, associated with selling x units of a particular product is given by the expression $0.001x^2 + 5x + 500$. The revenue from selling x units is given by $10x$.

35. Write a polynomial to represent the profit generated by the product.
36. Find the profit from sales of 1850 units.

Simplify.

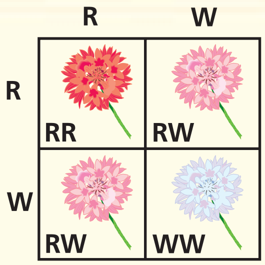
37. $(p + 6)(p - 4)$
38. $(a + 6)(a + 3)$
39. $(b + 5)(b - 5)$
40. $(6 - z)(6 + z)$
41. $(3x + 8)(2x + 6)$
42. $(4y - 6)(2y + 7)$
43. $(a^3 - b)(a^3 + b)$
44. $(m^2 - 5)(2m^2 + 3)$
45. $(x - 3y)^2$
46. $(1 + 4c)^2$
47. $d^{-3}(d^5 - 2d^3 + d^{-1})$
48. $x^{-3}y^2(yx^4 + y^{-1}x^3 + y^{-2}x^2)$
49. $(3b - c)^3$
50. $(x^2 + xy + y^2)(x - y)$

51. Simplify $(c^2 - 6cd - 2d^2) + (7c^2 - cd + 8d^2) - (-c^2 + 5cd - d^2)$.

52. Find the product of $6x - 5$ and $-3x + 2$.

53. **GENETICS** Suppose R and W represent two genes that a plant can inherit from its parents. The terms of the expansion of $(R + W)^2$ represent the possible pairings of the genes in the offspring. Write $(R + W)^2$ as a polynomial.

More About...



Genetics

The possible genes of parents and offspring can be summarized in a *Punnett square*, such as the one above.

Source: *Biology: The Dynamics of Life*

54. **CRITICAL THINKING** What is the degree of the product of a polynomial of degree 8 and a polynomial of degree 6? Include an example in support of your answer.

55. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can polynomials be applied to financial situations?

Include the following in your answer:
- an explanation of how a polynomial can be applied to a situation with a fixed percent rate of increase,
- two expressions in terms of r for the tuition in the fourth year, and
- an explanation of how to use one of the expressions and the 4% rate of increase to estimate Shenequa's tuition in the fourth year, and a comparison of the value you found to the value given in the table.

Standardized Test Practice

56. Which polynomial has degree 3?

(A) $x^3 + x^2 - 2x^4$
(B) $-2x^2 - 3x + 4$
(C) $x^2 + x + 12^3$
(D) $1 + x + x^3$

57. $(x + y) - (y + z) - (x + z) = ?$

(A) $2x + 2y + 2z$
(B) $-2z$
(C) $2y$
(D) $x - y - z$

Maintain Your Skills

Mixed Review

Simplify. Assume that no variable equals 0. *(Lesson 5-1)*

58. $(-4d^2)^3$
59. $5rt^2(2rt)^2$
60. $\frac{x^2yz^4}{xy^3z^2}$
61. $\left(\frac{3ab^2}{6a^2b}\right)^2$

62. Solve the system $4x - y = 0$, $2x + 3y = 14$ by using inverse matrices. *(Lesson 4-8)*

Graph each inequality. *(Lesson 2-7)*

63. $y \leq -\frac{1}{3}x + 2$
64. $x + y > -2$
65. $2x + y < 1$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify. Assume that no variable equals 0.**
*(To review **properties of exponents**, see Lesson 5-1.)*

66. $\frac{x^3}{x}$
67. $\frac{4y^5}{2y^2}$
68. $\frac{x^2y^3}{xy}$
69. $\frac{9a^3b}{3ab}$

5-3 Dividing Polynomials

What You'll Learn

- Divide polynomials using long division.
- Divide polynomials using synthetic division.

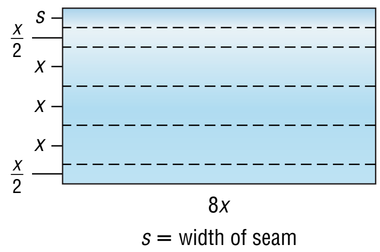
Vocabulary

- synthetic division

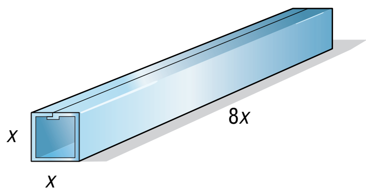
How can you use division of polynomials in manufacturing?

A machinist needed $32x^2 + x$ square inches of metal to make a square pipe $8x$ inches long. In figuring the area needed, she allowed a fixed amount of metal for overlap of the seam. If the width of the finished pipe will be x inches, how wide is the seam? You can use a quotient of polynomials to help find the answer.

Metal Needed



Finished Pipe



USE LONG DIVISION In Lesson 5-1, you learned to divide monomials. You can divide a polynomial by a monomial by using those same skills.

Example 1 Divide a Polynomial by a Monomial

Simplify $\dfrac{4x^3y^2 + 8xy^2 - 12x^2y^3}{4xy}$.

$$\frac{4x^3y^2 + 8xy^2 - 12x^2y^3}{4xy} = \frac{4x^3y^2}{4xy} + \frac{8xy^2}{4xy} - \frac{12x^2y^3}{4xy}$$ Sum of quotients

$$= \frac{4}{4} \cdot x^{3-1}y^{2-1} + \frac{8}{4} \cdot x^{1-1}y^{2-1} - \frac{12}{4} \cdot x^{2-1}y^{3-1}$$ Divide.

$$= x^2y + 2y - 3xy^2$$ $x^{1-1} = x^0$ or 1

You can use a process similar to long division to divide a polynomial by a polynomial with more than one term. The process is known as the *division algorithm*. When doing the division, remember that you can only add or subtract like terms.

Example 2 Division Algorithm

Use long division to find $(z^2 + 2z - 24) \div (z - 4)$.

$$\begin{array}{r} z \\ z-4\overline{)z^2+2z-24} \\ \underline{(-)z^2-4z} \\ 6z-24 \end{array}$$

$z(z - 4) = z^2 - 4z$

$2z - (-4z) = 6z$

$$\begin{array}{r} z+6 \\ z-4\overline{)z^2+2z-24} \\ \underline{(-)z^2-4z} \\ 6z-24 \\ \underline{(-)6z-24} \\ 0 \end{array}$$

The quotient is $z + 6$. The remainder is 0.

Just as with the division of whole numbers, the division of two polynomials may result in a quotient with a remainder. Remember that $9 \div 4 = 2 + \text{R}1$ and is often written as $2\frac{1}{4}$. The result of a division of polynomials with a remainder can be written in a similar manner.

Standardized Test Practice

A B C D

Example 3 Quotient with Remainder

Multiple-Choice Test Item

Which expression is equal to $(t^2 + 3t - 9)(5 - t)^{-1}$?

Ⓐ $t + 8 - \dfrac{31}{5 - t}$ Ⓑ $-t - 8$

Ⓒ $-t - 8 + \dfrac{31}{5 - t}$ Ⓓ $-t - 8 - \dfrac{31}{5 - t}$

Read the Test Item

Since the second factor has an exponent of -1, this is a division problem.

$$(t^2 + 3t - 9)(5 - t)^{-1} = \frac{t^2 + 3t - 9}{5 - t}$$

Solve the Test Item

$$\begin{array}{r l}
 & -t - 8 \\
-t + 5 & \overline{)\,t^2 + 3t - 9} \\
 & \underline{(-)t^2 - 5t} \\
 & \phantom{(-)t^2 -{}} 8t - 9 \\
 & \underline{(-)8t - 40} \\
 & \phantom{(-)t^2 - 8t - {}} 31
\end{array}$$

For ease in dividing, rewrite $5 - t$ as $-t + 5$.

$-t(-t + 5) = t^2 - 5t$

$3t - (-5t) = 8t$

$-8(-t + 5) = 8t - 40$

Subtract. $-9 - (-40) = 31$

The quotient is $-t - 8$, and the remainder is 31. Therefore,

$(t^2 + 3t - 9)(5 - t)^{-1} = -t - 8 + \frac{31}{5 - t}$. The answer is C.

Test-Taking Tip

You may be able to eliminate some of the answer choices by substituting the same value for t in the original expression and the answer choices and evaluating.

USE SYNTHETIC DIVISION

Synthetic division is a simpler process for dividing a polynomial by a binomial. Suppose you want to divide $5x^3 - 13x^2 + 10x - 8$ by $x - 2$ using long division. Compare the coefficients in this division with those in Example 4.

$$\begin{array}{r l}
 & 5x^2 - 3x + 4 \\
x - 2 & \overline{)\,5x^3 - 13x^2 + 10x - 8} \\
 & \underline{(-)5x^3 - 10x^2} \\
 & -3x^2 + 10x \\
 & \underline{(-)-3x^2 + 6x} \\
 & \phantom{(-)5x^3 - 13x^2 +{}} 4x - 8 \\
 & \underline{(-)4x - 8} \\
 & \phantom{(-)5x^3 - 13x^2 + 10x - {}} 0
\end{array}$$

Example 4 Synthetic Division

Use synthetic division to find $(5x^3 - 13x^2 + 10x - 8) \div (x - 2)$.

Step 1 Write the terms of the dividend so that the degrees of the terms are in descending order. Then write just the coefficients as shown at the right.

$$\begin{array}{cccc}
5x^3 & -13x^2 & +10x & -8 \\
\downarrow & \downarrow & \downarrow & \downarrow \\
5 & -13 & 10 & -8
\end{array}$$

Step 2 Write the constant r of the divisor $x - r$ to the left. In this case, $r = 2$. Bring the first coefficient, 5, down as shown.

$$\begin{array}{c|cccc}
2 & 5 & -13 & 10 & -8 \\
 & & & & \\
\hline
 & 5 & & &
\end{array}$$

Step 3 Multiply the first coefficient by r: $2 \cdot 5 = 10$. Write the product under the second coefficient. Then add the product and the second coefficient: $-13 + 10 = -3$.

2	5	−13	10	−8
		10		
	5	−3		

Step 4 Multiply the sum, -3, by r: $2(-3) = -6$. Write the product under the next coefficient and add: $10 + (-6) = 4$.

2	5	−13	10	−8
		10	−6	
	5	−3	4	

Step 5 Multiply the sum, 4, by r: $2 \cdot 4 = 8$. Write the product under the next coefficient and add: $-8 + 8 = 0$. The remainder is 0.

2	5	−13	10	−8
		10	−6	8
	5	−3	4	0

The numbers along the bottom row are the coefficients of the quotient. Start with the power of x that is one less than the degree of the dividend. Thus, the quotient is $5x^2 - 3x + 4$.

To use synthetic division, the divisor must be of the form $x - r$. If the coefficient of x in a divisor is not 1, you can rewrite the division expression so that you can use synthetic division.

Example 5 Divisor with First Coefficient Other than 1

Use synthetic division to find $(8x^4 - 4x^2 + x + 4) \div (2x + 1)$.

Use division to rewrite the divisor so it has a first coefficient of 1.

$$\frac{8x^4 - 4x^2 + x + 4}{2x + 1} = \frac{(8x^4 - 4x^2 + x + 4) \div 2}{(2x + 1) \div 2}$$ Divide numerator and denominator by 2.

$$= \frac{4x^4 - 2x^2 + \frac{1}{2}x + 2}{x + \frac{1}{2}}$$ Simplify the numerator and denominator.

Since the numerator does not have an x^3-term, use a coefficient of 0 for x^3.

$x - r = x + \frac{1}{2}$, so $r = -\frac{1}{2}$.

$-\frac{1}{2}$	4	0	−2	$\frac{1}{2}$	2
		−2	1	$\frac{1}{2}$	$-\frac{1}{2}$
	4	−2	−1	1	$\frac{3}{2}$

The result is $4x^3 - 2x^2 - x + 1 + \frac{\frac{3}{2}}{x + \frac{1}{2}}$. Now simplify the fraction.

$$\frac{\frac{3}{2}}{x + \frac{1}{2}} = \frac{3}{2} \div \left(x + \frac{1}{2}\right)$$ Rewrite as a division expression.

$$= \frac{3}{2} \div \frac{2x + 1}{2}$$ $x + \frac{1}{2} = \frac{2x}{2} + \frac{1}{2} = \frac{2x + 1}{2}$

$$= \frac{3}{2} \cdot \frac{2}{2x + 1}$$ Multiply by the reciprocal.

$$= \frac{3}{2x + 1}$$ Multiply.

The solution is $4x^3 - 2x^2 - x + 1 + \frac{3}{2x + 1}$.

(continued on the next page)

CHECK Divide using long division.

$$\begin{array}{r} 4x^3 - 2x^2 - x + 1 \\ 2x + 1\overline{)8x^4 + 0x^3 - 4x^2 + x + 4} \\ \underline{(-)8x^4 + 4x^3} \qquad\qquad\qquad \\ -4x^3 - 4x^2 \qquad\qquad \\ \underline{(-)-4x^3 - 2x^2} \qquad\qquad \\ -2x^2 + x \qquad \\ \underline{(-)-2x^2 - x} \qquad \\ 2x + 4 \\ \underline{(-)2x + 1} \\ 3 \end{array}$$

The result is $4x^3 - 2x^2 - x + 1 + \frac{3}{2x + 1}$. ✓

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a quotient of two polynomials such that the remainder is 5.
2. **Explain** why synthetic division cannot be used to simplify $\frac{x^3 - 3x + 1}{x^2 + 1}$.
3. **FIND THE ERROR** Shelly and Jorge are dividing $x^3 - 2x^2 + x - 3$ by $x - 4$.

Shelly

4	1	-2	1	-3
		4	-24	100
	1	-6	25	-103

Jorge

4	1	-2	1	-3
		4	8	36
	1	2	9	33

Who is correct? Explain your reasoning.

Guided Practice

Simplify.

4. $\frac{6xy^2 - 3xy + 2x^2y}{xy}$
5. $(5ab^2 - 4ab + 7a^2b)(ab)^{-1}$
6. $(x^2 - 10x - 24) \div (x + 2)$
7. $(3a^4 - 6a^3 - 2a^2 + a - 6) \div (a + 1)$
8. $(z^5 - 3z^2 - 20) \div (z - 2)$
9. $(x^3 + y^3) \div (x + y)$
10. $\frac{x^3 + 13x^2 - 12x - 8}{x + 2}$
11. $(b^4 - 2b^3 + b^2 - 3b + 2)(b - 2)^{-1}$
12. $(12y^2 + 36y + 15) \div (6y + 3)$
13. $\frac{9b^2 + 9b - 10}{3b - 2}$

Standardized Test Practice

14. Which expression is equal to $(x^2 - 4x + 6)(x - 3)^{-1}$?

   (A) $x - 1$
   (B) $x - 1 + \frac{3}{x - 3}$
   (C) $x - 1 - \frac{3}{x - 3}$
   (D) $-x + 1 - \frac{3}{x - 3}$

Practice and Apply

Simplify.

15. $\frac{9a^3b^2 - 18a^2b^3}{3a^2b}$
16. $\frac{5xy^2 - 6y^3 + 3x^2y^3}{xy}$
17. $(28c^3d - 42cd^2 + 56cd^3) \div (14cd)$
18. $(12mn^3 + 9m^2n^2 - 15m^2n) \div (3mn)$
19. $(2y^3z + 4y^2z^2 - 8y^4z^5)(yz)^{-1}$
20. $(a^3b^2 - a^2b + 2a)(-ab)^{-1}$

Homework Help

For Exercises	See Examples
15–20, 51	1
21–34, 49, 50, 52–54	2, 4
35–38	3, 4
39–48	2, 3, 5

Extra Practice
See page 837.

21. $(b^3 + 8b^2 - 20b) \div (b - 2)$
22. $(x^2 - 12x - 45) \div (x + 3)$
23. $(n^3 + 2n^2 - 5n + 12) \div (n + 4)$
24. $(2c^3 - 3c^2 + 3c - 4) \div (c - 2)$
25. $(x^4 - 3x^3 + x^2 - 5) \div (x + 2)$
26. $(6w^5 - 18w^2 - 120) \div (w - 2)$
27. $(x^3 - 4x^2) \div (x - 4)$
28. $(x^3 - 27) \div (x - 3)$
29. $\dfrac{y^3 + 3y^2 - 5y - 4}{y + 4}$
30. $\dfrac{m^3 + 3m^2 - 7m - 21}{m + 3}$
31. $\dfrac{a^4 - 5a^3 - 13a^2 + 10}{a + 1}$
32. $\dfrac{2m^4 - 5m^3 - 10m + 8}{m - 3}$
33. $\dfrac{x^5 - 7x^3 + x + 1}{x + 3}$
34. $\dfrac{3c^5 + 5c^4 + c + 5}{c + 2}$
35. $(g^2 + 8g + 15)(g + 3)^{-1}$
36. $(2b^3 + b^2 - 2b + 3)(b + 1)^{-1}$
37. $(t^5 - 3t^2 - 20)(t - 2)^{-1}$
38. $(y^5 + 32)(y + 2)^{-1}$
39. $(6t^3 + 5t^2 + 9) \div (2t + 3)$
40. $(2h^3 - 5h^2 + 22h) \div (2h + 3)$
41. $\dfrac{9d^3 + 5d - 8}{3d - 2}$
42. $\dfrac{4x^3 + 5x^2 - 3x - 1}{4x + 1}$
43. $\dfrac{2x^4 + 3x^3 - 2x^2 - 3x - 6}{2x + 3}$
44. $\dfrac{6x^4 + 5x^3 + x^2 - 3x + 1}{3x + 1}$
45. $\dfrac{x^3 - 3x^2 + x - 3}{x^2 + 1}$
46. $\dfrac{x^4 + x^2 - 3x + 5}{x^2 + 2}$
47. $\dfrac{x^3 + 3x^2 + 3x + 2}{x^2 + x + 1}$
48. $\dfrac{x^3 - 4x^2 + 5x - 6}{x^2 - x + 2}$

49. What is $x^3 - 2x^2 + 4x - 3$ divided by $x - 1$?

50. Divide $2y^3 + y^2 - 5y + 2$ by $y + 2$.

51. **BUSINESS** A company estimates that it costs $0.03x^2 + 4x + 1000$ dollars to produce x units of a product. Find an expression for the average cost per unit.

52. **ENTERTAINMENT** A magician gives these instructions to a volunteer.
- Choose a number and multiply it by 3.
- Then add the sum of your number and 8 to the product you found.
- Now divide by the sum of your number and 2.

What number will the volunteer always have at the end? Explain.

Career Choices

Cost Analyst

Cost analysts study and write reports about the factors involved in the cost of production.

Online Research For information about a career in cost analysis, visit: www.algebra2.com/careers

MEDICINE **For Exercises 53 and 54, use the following information.**
The number of students at a large high school who will catch the flu during an outbreak can be estimated by $n = \dfrac{170t^2}{t^2 + 1}$, where t is the number of weeks from the beginning of the epidemic and n is the number of ill people.

53. Perform the division indicated by $\dfrac{170t^2}{t^2 + 1}$.

54. Use the formula to estimate how many people will become ill during the first week.

PHYSICS **For Exercises 55–57, suppose an object moves in a straight line so that after t seconds, it is $t^3 + t^2 + 6t$ feet from its starting point.**

55. Find the distance the object travels between the times $t = 2$ and $t = x$.

56. How much time elapses between $t = 2$ and $t = x$?

57. Find a simplified expression for the average speed of the object between times $t = 2$ and $t = x$.

58. **CRITICAL THINKING** Suppose the result of dividing one polynomial by another is $r^2 - 6r + 9 - \dfrac{1}{r - 3}$. What two polynomials might have been divided?

59. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can you use division of polynomials in manufacturing?

Include the following in your answer:
- the dimensions of the piece of metal that the machinist needs,
- the formula from geometry that applies to this situation, and
- an explanation of how to use division of polynomials to find the width s of the seam.

Standardized Test Practice

60. An office employs x women and 3 men. What is the ratio of the total number of employees to the number of women?

Ⓐ $1 + \frac{3}{x}$ Ⓑ $\frac{x}{x+3}$ Ⓒ $\frac{3}{x}$ Ⓓ $\frac{x}{3}$

61. If $a + b = c$ and $a = b$, then all of the following are true EXCEPT

Ⓐ $a - c = b - c$. Ⓑ $a - b = 0$.

Ⓒ $2a + 2b = 2c$. Ⓓ $c - b = 2a$.

Maintain Your Skills

Mixed Review

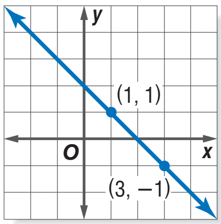
Simplify. *(Lesson 5-2)*

62. $(2x^2 - 3x + 5) - (3x^2 + x - 9)$
63. $y^2z(y^2z^3 - yz^2 + 3)$
64. $(y + 5)(y - 3)$
65. $(a - b)^2$

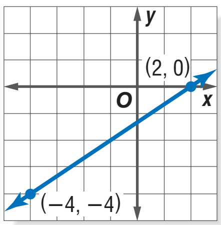
66. **ASTRONOMY** Earth is an average of 1.5×10^{11} meters from the Sun. Light travels at 3×10^8 meters per second. About how long does it take sunlight to reach Earth? *(Lesson 5-1)*

Write an equation in slope-intercept form for each graph. *(Lesson 2-4)*

67.



68.



Getting Ready for the Next Lesson

BASIC SKILL Find the greatest common factor of each set of numbers.

69. 18, 27
70. 24, 84
71. 16, 28
72. 12, 27, 48
73. 12, 30, 54
74. 15, 30, 65

Practice Quiz 1

Lessons 5-1 through 5-3

Express each number in scientific notation. *(Lesson 5-1)*

1. 653,000,000
2. 0.0072

Simplify. *(Lessons 5-1 and 5-2)*

3. $(-3x^2y)^3(2x)^2$
4. $\frac{a^6b^{-2}c}{a^3b^2c^4}$
5. $\left(\frac{x^2z}{xz^4}\right)^2$
6. $(9x + 2y) - (7x - 3y)$
7. $(t + 2)(3t - 4)$
8. $(n + 2)(n^2 - 3n + 1)$

Simplify. *(Lesson 5-3)*

9. $(m^3 - 4m^2 - 3m - 7) \div (m - 4)$
10. $\frac{2d^3 - d^2 - 9d + 9}{2d - 3}$

CONTENTS

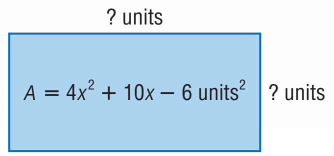
5-4 Factoring Polynomials

What You'll Learn

- Factor polynomials.
- Simplify polynomial quotients by factoring.

How does factoring apply to geometry?

Suppose the expression $4x^2 + 10x - 6$ represents the area of a rectangle. Factoring can be used to find possible dimensions of the rectangle.



FACTOR POLYNOMIALS Whole numbers are factored using prime numbers. For example, $100 = 2 \cdot 2 \cdot 5 \cdot 5$. Many polynomials can also be factored. Their factors, however, are other polynomials. Polynomials that cannot be factored are called *prime*.

The table below summarizes the most common factoring techniques used with polynomials.

Concept Summary — *Factoring Techniques*

Number of Terms	Factoring Technique	General Case
any number	Greatest Common Factor (GCF)	$a^3b^2 + 2a^2b - 4ab^2 = ab(a^2b + 2a - 4b)$
two	Difference of Two Squares Sum of Two Cubes Difference of Two Cubes	$a^2 - b^2 = (a + b)(a - b)$ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
three	Perfect Square Trinomials	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$
	General Trinomials	$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$
four or more	Grouping	$ax + bx + ay + by = x(a + b) + y(a + b)$ $= (a + b)(x + y)$

Whenever you factor a polynomial, always look for a common factor first. Then determine whether the resulting polynomial factor can be factored again using one or more of the methods listed in the table above.

Example 1 GCF

Factor $6x^2y^2 - 2xy^2 + 6x^3y$.

$$6x^2y^2 - 2xy^2 + 6x^3y = (2 \cdot 3 \cdot x \cdot x \cdot y \cdot y) - (2 \cdot x \cdot y \cdot y) + (2 \cdot 3 \cdot x \cdot x \cdot x \cdot y)$$

$$= (2xy \cdot 3xy) - (2xy \cdot y) + (2xy \cdot 3x^2)$$

$$= 2xy(3xy - y + 3x^2)$$

The GCF is $2xy$. The remaining polynomial cannot be factored using the methods above.

Check this result by finding the product.

A GCF is also used in grouping to factor a polynomial of four or more terms.

Example 2 Grouping

Factor $a^3 - 4a^2 + 3a - 12$.

$$
\begin{aligned}
a^3 - 4a^2 + 3a - 12 &= (a^3 - 4a^2) + (3a - 12) && \text{Group to find a GCF.}\\
&= a^2(a - 4) + 3(a - 4) && \text{Factor the GCF of each binomial.}\\
&= (a - 4)(a^2 + 3) && \text{Distributive Property}
\end{aligned}
$$

You can use algebra tiles to model factoring a polynomial.

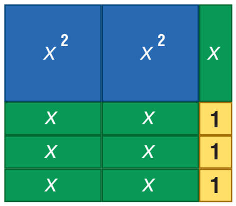
Algebra Activity

Factoring Trinomials

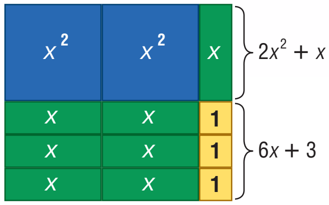
Use algebra tiles to factor $2x^2 + 7x + 3$.

Model and Analyze

- Use algebra tiles to model $2x^2 + 7x + 3$.
- To find the product that resulted in this polynomial, arrange the tiles to form a rectangle.



- Notice that the total area can be expressed as the sum of the areas of two smaller rectangles.



Use these expressions to rewrite the trinomial. Then factor.

$$
\begin{aligned}
2x^2 + 7x + 3 &= (2x^2 + x) + (6x + 3) && \text{total area = sum of areas of smaller rectangles}\\
&= x(2x + 1) + 3(2x + 1) && \text{Factor out each GCF.}\\
&= (2x + 1)(x + 3) && \text{Distributive Property}
\end{aligned}
$$

Make a Conjecture

Study the factorization of $2x^2 + 7x + 3$ above.

1. What are the coefficients of the two x terms in $(2x^2 + x) + (6x + 3)$? Find their sum and their product.
2. Compare the sum you found in Exercise 1 to the coefficient of the x term in $2x^2 + 7x + 3$.
3. Find the product of the coefficient of the x^2 term and the constant term in $2x^2 + 7x + 3$. How does it compare to the product in Exercise 1?
4. Make a conjecture about how to factor $3x^2 + 7x + 2$.

Study Tip

Algebra Tiles

When modeling a polynomial with algebra tiles, it is easiest to arrange the x^2 tiles first, then the x tiles and finally the 1 tiles to form a rectangle.

The FOIL method can help you factor a polynomial into the product of two binomials. Study the following example.

$$
\begin{aligned}
(ax + b)(cx + d) &= \overbrace{ax \cdot cx}^{F} + \overbrace{ax \cdot d}^{O} + \overbrace{b \cdot cx}^{I} + \overbrace{b \cdot d}^{L}\\
&= acx^2 + (ad + bc)x + bd
\end{aligned}
$$

Notice that the product of the coefficient of x^2 and the constant term is $abcd$. The product of the two terms in the coefficient of x is also $abcd$.

Example 3 Two or Three Terms

Factor each polynomial.

a. $5x^2 - 13x + 6$

To find the coefficients of the x-terms, you must find two numbers whose product is $5 \cdot 6$ or 30, and whose sum is -13. The two coefficients must be -10 and -3 since $(-10)(-3) = 30$ and $-10 + (-3) = -13$.

Rewrite the expression using $-10x$ and $-3x$ in place of $-13x$ and factor by grouping.

$5x^2 - 13x + 6 = 5x^2 - 10x - 3x + 6$ Substitute $-10x - 3x$ for $-13x$.

$= (5x^2 - 10x) + (-3x + 6)$ Associative Property

$= 5x(x - 2) - 3(x - 2)$ Factor out the GCF of each group.

$= (5x - 3)(x - 2)$ Distributive Property

b. $3xy^2 - 48x$

$3xy^2 - 48x = 3x(y^2 - 16)$ Factor out the GCF.

$= 3x(y + 4)(y - 4)$ $y^2 - 16$ is the difference of two squares.

c. $c^3d^3 + 27$

$c^3d^3 = (cd)^3$ and $27 = 3^3$. Thus, this is the sum of two cubes.

$c^3d^3 + 27 = (cd + 3)[(cd)^2 - 3(cd) + 3^2]$ Sum of two cubes formula with $a = cd$ and $b = 3$

$= (cd + 3)(c^2d^2 - 3cd + 9)$ Simplify.

d. $m^6 - n^6$

This polynomial could be considered the difference of two squares or the difference of two cubes. The difference of two squares should always be done before the difference of two cubes. This will make the next step of the factorization easier.

$m^6 - n^6 = (m^3 + n^3)(m^3 - n^3)$ Difference of two squares

$= (m + n)(m^2 - mn + n^2)(m - n)(m^2 + mn + n^2)$ Sum and difference of two cubes

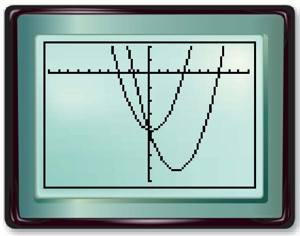
You can use a graphing calculator to check that the factored form of a polynomial is correct.

Graphing Calculator Investigation

Factoring Polynomials

Is the factored form of $2x^2 - 11x - 21$ equal to $(2x - 7)(x + 3)$? You can find out by graphing $y = 2x^2 - 11x - 21$ and $y = (2x - 7)(x + 3)$. If the two graphs coincide, the factored form is probably correct.

- Enter $y = 2x^2 - 11x - 21$ and $y = (2x - 7)(x + 3)$ on the Y= screen.
- Graph the functions. Since two different graphs appear, $2x^2 - 11x - 21 \neq (2x - 7)(x + 3)$.



[−10, 10] scl: 1 by [−40, 10] scl: 5

Think and Discuss

1. Determine if $x^2 + 5x - 6 = (x - 3)(x - 2)$ is a true statement. If not, write the correct factorization.
2. Does this method guarantee a way to check the factored form of a polynomial? Why or why not?

SIMPLIFY QUOTIENTS In Lesson 5-3, you learned to simplify the quotient of two polynomials by using long division or synthetic division. Some quotients can be simplified using factoring.

Example 4 Quotient of Two Trinomials

Simplify $\frac{x^2 + 2x - 3}{x^2 + 7x + 12}$.

$\frac{x^2 + 2x - 3}{x^2 + 7x + 12} = \frac{\overset{1}{\cancel{(x + 3)}}(x - 1)}{(x + 4)\underset{1}{\cancel{(x + 3)}}}$ Factor the numerator and denominator.

$= \frac{x - 1}{x + 4}$ Divide. Assume $x \neq -3, -4$.

Therefore, $\frac{x^2 + 2x - 3}{x^2 + 7x + 12} = \frac{x - 1}{x + 4}$, if $x \neq -3, -4$.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write an example of a perfect square trinomial.
2. **Find a counterexample** to the statement $a^2 + b^2 = (a + b)^2$.
3. **Decide** whether the statement $\frac{x - 2}{x^2 + x - 6} = \frac{1}{x + 3}$ is *sometimes*, *always*, or *never* true.

Guided Practice

Factor completely. If the polynomial is not factorable, write *prime*.

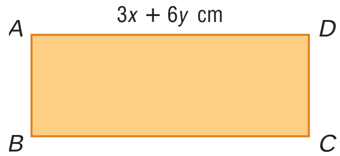
4. $-12x^2 - 6x$
5. $a^2 + 5a + ab$
6. $21 - 7y + 3x - xy$
7. $y^2 - 6y + 8$
8. $z^2 - 4z - 12$
9. $3b^2 - 48$
10. $16w^2 - 169$
11. $h^3 + 8000$

Simplify. Assume that no denominator is equal to 0.

12. $\frac{x^2 - 2x - 8}{x^2 - 5x - 14}$
13. $\frac{2y^2 + 8y}{y^2 - 16}$

Application

14. **GEOMETRY** Find the width of rectangle *ABCD* if its area is $3x^2 + 9xy + 6y^2$ square centimeters.



Practice and Apply

Factor completely. If the polynomial is not factorable, write *prime*.

15. $2xy^3 - 10x$
16. $6a^2b^2 + 18ab^3$
17. $12cd^3 - 8c^2d^2 + 10c^5d^3$
18. $3a^2bx + 15cx^2y + 25ad^3y$
19. $8yz - 6z - 12y + 9$
20. $3ax - 15a + x - 5$
21. $x^2 + 7x + 6$
22. $y^2 - 5y + 4$
23. $2a^2 + 3a + 1$
24. $2b^2 + 13b - 7$
25. $6c^2 + 13c + 6$
26. $12m^2 - m - 6$
27. $3n^2 + 21n - 24$
28. $3z^2 + 24z + 45$

Homework Help

For Exercises	See Examples
15–18	1
19, 20	2
21–38, 43–45, 55	3
39–42	2, 3
46–54	4

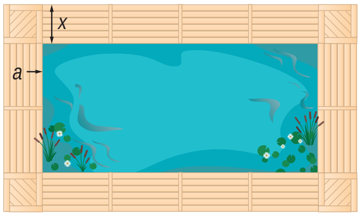
Extra Practice
See page 837.

29. $x^2 + 12x + 36$
30. $x^2 - 6x + 9$
31. $16a^2 + 25b^2$
32. $3m^2 - 3n^2$
33. $y^4 - z^2$
34. $3x^2 - 27y^2$
35. $z^3 + 125$
36. $t^3 - 8$
37. $p^4 - 1$
38. $x^4 - 81$
39. $7ac^2 + 2bc^2 - 7ad^2 - 2bd^2$
40. $8x^2 + 8xy + 8xz + 3x + 3y + 3z$
41. $5a^2x + 4aby + 3acz - 5abx - 4b^2y - 3bcz$
42. $3a^3 + 2a^2 - 5a + 9a^2b + 6ab - 15b$

43. Find the factorization of $3x^2 + x - 2$.

44. What are the factors of $2y^2 + 9y + 4$?

45. **LANDSCAPING** A boardwalk that is x feet wide is built around a rectangular pond. The combined area of the pond and the boardwalk is $4x^2 + 140x + 1200$ square feet. What are the dimensions of the pond?



Simplify. Assume that no denominator is equal to 0.

46. $\dfrac{x^2 + 4x + 3}{x^2 - x - 12}$
47. $\dfrac{x^2 + 4x - 5}{x^2 - 7x + 6}$
48. $\dfrac{x^2 - 25}{x^2 + 3x - 10}$
49. $\dfrac{x^2 - 6x + 8}{x^3 - 8}$
50. $\dfrac{x^2}{(x^2 - x)(x - 1)^{-1}}$
51. $\dfrac{x + 1}{(x^2 + 3x + 2)(x + 2)^{-2}}$

BUILDINGS **For Exercises 52 and 53, use the following information.**
When an object is dropped from a tall building, the distance it falls between 1 second after it is dropped and x seconds after it is dropped is $16x^2 - 16$ feet.

52. How much time elapses between 1 second after it is dropped and x seconds after it is dropped?
53. What is the average speed of the object during that time period?

54. **GEOMETRY** The length of one leg of a right triangle is $x - 6$ centimeters, and the area is $\frac{1}{2}x^2 - 7x + 24$ square centimeters. What is the length of the other leg?

55. **CRITICAL THINKING** Factor $64p^{2n} + 16p^n + 1$.

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How does factoring apply to geometry?

Include the following in your answer:

- an explanation of how to use factoring to find possible dimensions for the rectangle described at the beginning of the lesson, and
- why your dimensions are not the only ones possible, even if you assume that the dimensions are binomials with integer coefficients.

More About. . .

Buildings

The tallest buildings in the world are the Petronas Towers in Kuala Lumpur, Malaysia. Each is 1483 feet tall.

Source: www.worldstallest.com

Standardized Test Practice

57. Which of the following is the factorization of $2x - 15 + x^2$?

(A) $(x - 3)(x - 5)$ (B) $(x - 3)(x + 5)$

(C) $(x + 3)(x - 5)$ (D) $(x + 3)(x + 5)$

58. Which is not a factor of $x^3 - x^2 - 2x$?

(A) x (B) $x + 1$ (C) $x - 1$ (D) $x - 2$

Graphing Calculator

CHECK FACTORING **Use a graphing calculator to determine if each polynomial is factored correctly. Write *yes* or *no*. If the polynomial is not factored correctly, find the correct factorization.**

59. $3x^2 + 5x + 2 \stackrel{?}{=} (3x + 2)(x + 1)$

60. $x^3 + 8 \stackrel{?}{=} (x + 2)(x^2 - x + 4)$

61. $2x^2 - 5x - 3 \stackrel{?}{=} (x - 1)(2x + 3)$

62. $3x^2 - 48 \stackrel{?}{=} 3(x + 4)(x - 4)$

Maintain Your Skills

Mixed Review

Simplify. *(Lesson 5-3)*

63. $(t^3 - 3t + 2) \div (t + 2)$

64. $(y^2 + 4y + 3)(y + 1)^{-1}$

65. $\dfrac{x^3 - 3x^2 + 2x - 6}{x - 3}$

66. $\dfrac{3x^4 + x^3 - 8x^2 + 10x - 3}{3x - 2}$

Simplify. *(Lesson 5-2)*

67. $(3x^2 - 2xy + y^2) + (x^2 + 5xy - 4y^2)$

68. $(2x + 4)(7x - 1)$

Perform the indicated operations, if possible. *(Lesson 4-5)*

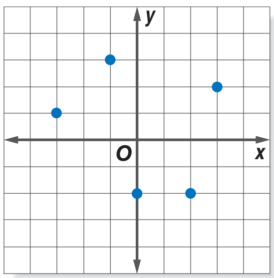
69. $[3 \quad -1] \cdot \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

70. $\begin{bmatrix} 1 & -4 \\ 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 \\ 9 & -1 \end{bmatrix}$

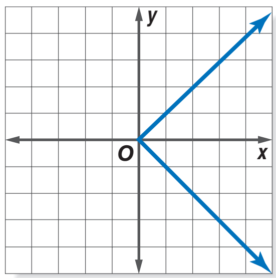
71. **PHOTOGRAPHY** The perimeter of a rectangular picture is 86 inches. Twice the width exceeds the length by 2 inches. What are the dimensions of the picture? *(Lesson 3-2)*

Determine whether each relation is a function. Write *yes* or *no*. *(Lesson 2-1)*

72.



73.



State the property illustrated by each equation. *(Lesson 1-2)*

74. $(3 + 8)5 = 3(5) + 8(5)$

75. $1 + (7 + 4) = (1 + 7) + 4$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Determine whether each number is *rational* or *irrational*.**
*(To review **rational and irrational numbers**, see Lesson 1-2.)*

76. 4.63

77. π

78. $\dfrac{16}{3}$

79. 8.333…

80. 7.323223222…

81. $9.7\overline{1}$

5-5 Roots of Real Numbers

What You'll Learn

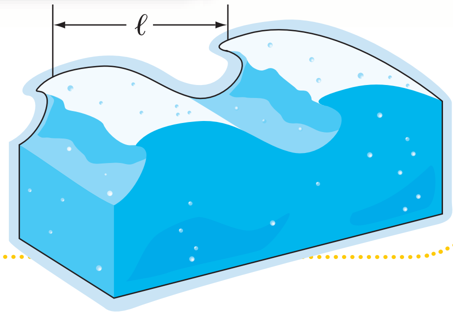
- Simplify radicals.
- Use a calculator to approximate radicals.

Vocabulary

- square root
- *n*th root
- principal root

How do square roots apply to oceanography?

The speed s in knots of a wave can be estimated using the formula $s = 1.34\sqrt{\ell}$, where ℓ is the length of the wave in feet. This is an example of an equation that contains a square root.



SIMPLIFY RADICALS Finding the square root of a number and squaring a number are inverse operations. To find the **square root** of a number n, you must find a number whose square is n. For example, 7 is a square root of 49 since $7^2 = 49$. Since $(-7)^2 = 49$, -7 is also a square root of 49.

Key Concept — Definition of Square Root

- **Words** For any real numbers a and b, if $a^2 = b$, then a is a square root of b.
- **Example** Since $5^2 = 25$, 5 is a square root of 25.

Since finding the square root of a number and squaring a number are inverse operations, it makes sense that the inverse of raising a number to the nth power is finding the ***n*th root** of a number. The table below shows the relationship between raising a number to a power and taking that root of a number.

Powers	Factors	Roots
$a^3 = 125$	$5 \cdot 5 \cdot 5 = 125$	5 is a cube root of 125.
$a^4 = 81$	$3 \cdot 3 \cdot 3 \cdot 3 = 81$	3 is a fourth root of 81.
$a^5 = 32$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$	2 is a fifth root of 32.
$a^n = b$	$\underbrace{a \cdot a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors of } a} = b$	a is an nth root of b.

This pattern suggests the following formal definition of an nth root.

Key Concept — Definition of nth Root

- **Words** For any real numbers a and b, and any positive integer n, if $a^n = b$, then a is an nth root of b.
- **Example** Since $2^5 = 32$, 2 is a fifth root of 32.

Study Tip

Reading Math
$\sqrt[n]{50}$ is read *the nth root of 50.*

The symbol $\sqrt[n]{\ }$ indicates an nth root.

$$\sqrt[n]{50}$$

index → n; radical sign → $\sqrt{\ }$; radicand → 50

Some numbers have more than one real nth root. For example, 36 has two square roots, 6 and -6. When there is more than one real root, the nonnegative root is called the **principal root**. When no index is given, as in $\sqrt{36}$, the radical sign indicates the principal square root. The symbol $\sqrt[n]{b}$ stands for the principal nth root of b. If n is odd and b is negative, there will be no nonnegative root. In this case, the principal root is negative.

$\sqrt{16} = 4$	$\sqrt{16}$ indicates the principal square root of 16.
$-\sqrt{16} = -4$	$-\sqrt{16}$ indicates the opposite of the principal square root of 16.
$\pm\sqrt{16} = \pm 4$	$\pm\sqrt{16}$ indicates both square roots of 16. $\pm$ means positive or negative.
$\sqrt[3]{-125} = -5$	$\sqrt[3]{-125}$ indicates the principal cube root of -125.
$-\sqrt[4]{81} = -3$	$-\sqrt[4]{81}$ indicates the opposite of the principal fourth root of 81.

Concept Summary — *Real nth roots of b, $\sqrt[n]{b}$, or $-\sqrt[n]{b}$*

n	$\sqrt[n]{b}$ if $b > 0$	$\sqrt[n]{b}$ if $b < 0$	$b = 0$
even	one positive root, one negative root $\pm\sqrt[4]{625} = \pm 5$	no real roots $\sqrt{-4}$ is not a real number.	one real root, 0 $\sqrt[n]{0} = 0$
odd	one positive root, no negative roots $\sqrt[3]{8} = 2$	no positive roots, one negative root $\sqrt[5]{-32} = -2$	

Example 1 Find Roots

Simplify.

a. $\pm\sqrt{25x^4}$

$$\pm\sqrt{25x^4} = \pm\sqrt{(5x^2)^2}$$
$$= \pm 5x^2$$

The square roots of $25x^4$ are $\pm 5x^2$.

b. $-\sqrt{(y^2 + 2)^8}$

$$-\sqrt{(y^2+2)^8} = -\sqrt{[(y^2+2)^4]^2}$$
$$= -(y^2+2)^4$$

The opposite of the principal square root of $(y^2 + 2)^8$ is $-(y^2 + 2)^4$.

c. $\sqrt[5]{32x^{15}y^{20}}$

$$\sqrt[5]{32x^{15}y^{20}} = \sqrt[5]{(2x^3y^4)^5}$$
$$= 2x^3y^4$$

The principal fifth root of $32x^{15}y^{20}$ is $2x^3y^4$.

d. $\sqrt{-9}$

$$\sqrt{-9} = \sqrt[2]{-9}$$

n is even. b is negative.

Thus, $\sqrt{-9}$ is not a real number.

When you find the nth root of an even power and the result is an odd power, you must take the absolute value of the result to ensure that the answer is nonnegative.

$$\sqrt{(-5)^2} = |-5| \text{ or } 5 \qquad \sqrt{(-2)^6} = |(-2)^3| \text{ or } 8$$

If the result is an even power or you find the nth root of an odd power, there is no need to take the absolute value. *Why?*

Example 2 Simplify Using Absolute Value

Simplify.

a. $\sqrt[8]{x^8}$

Note that x is an eighth root of x^8. The index is even, so the principal root is nonnegative. Since x could be negative, you must take the absolute value of x to identify the principal root.

$\sqrt[8]{x^8} = |x|$

b. $\sqrt[4]{81(a+1)^{12}}$

$\sqrt[4]{81(a+1)^{12}} = \sqrt[4]{[3(a+1)^3]^4}$

Since the index 4 is even and the exponent 3 is odd, you must use the absolute value of $(a+1)^3$.

$\sqrt[4]{81(a+1)^{12}} = 3|(a+1)^3|$

APPROXIMATE RADICALS WITH A CALCULATOR Recall that real numbers that cannot be expressed as terminating or repeating decimals are *irrational numbers*. $\sqrt{2}$ and $\sqrt{3}$ are examples of irrational numbers. Decimal approximations for irrational numbers are often used in applications.

Example 3 Approximate a Square Root

PHYSICS **The time T in seconds that it takes a pendulum to make a complete swing back and forth is given by the formula $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum in feet and g is the acceleration due to gravity, 32 feet per second squared. Find the value of T for a 3-foot-long pendulum.**

Explore You are given the values of L and g and must find the value of T. Since the units on g are feet per second squared, the units on the time T should be seconds.

Plan Substitute the values for L and g into the formula. Use a calculator to evaluate.

Solve

$T = 2\pi\sqrt{\frac{L}{g}}$ Original formula

$= 2\pi\sqrt{\frac{3}{32}}$ $L = 3, g = 32$

≈ 1.92 Use a calculator.

It takes the pendulum about 1.92 seconds to make a complete swing.

Examine The closest square to $\frac{3}{32}$ is $\frac{1}{9}$, and π is approximately 3, so the answer should be close to $2(3)\sqrt{\frac{1}{9}} = 2(3)\left(\frac{1}{3}\right)$ or 2. The answer is reasonable.

Study Tip

Graphing Calculators

To find a root of index greater than 2, first type the index. Then select $\sqrt[x]{\ }$ from the MATH menu. Finally, enter the radicand.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a number whose principal square root and cube root are both integers.
2. **Explain** why it is not always necessary to take the absolute value of a result to indicate the principal root.
3. **Determine** whether the statement $\sqrt[4]{(-x)^4} = x$ is *sometimes*, *always*, or *never* true. Explain your reasoning.

Guided Practice

Use a calculator to approximate each value to three decimal places.

4. $\sqrt{77}$ 5. $-\sqrt[3]{19}$ 6. $\sqrt[4]{48}$

Simplify.

7. $\sqrt[3]{64}$ 8. $\sqrt{(-2)^2}$ 9. $\sqrt[5]{-243}$ 10. $\sqrt[4]{-4096}$

11. $\sqrt[3]{x^3}$ 12. $\sqrt[4]{y^4}$ 13. $\sqrt{36a^2b^4}$ 14. $\sqrt{(4x + 3y)^2}$

Application

15. **OPTICS** The distance D in miles from an observer to the horizon over flat land or water can be estimated using the formula $D = 1.23\sqrt{h}$, where h is the height in feet of the point of observation. How far is the horizon for a person whose eyes are 6 feet above the ground?

Practice and Apply

Homework Help

For Exercises	See Examples
16–27, 60–62	3
28–59	1, 2

Extra Practice
See page 838.

Use a calculator to approximate each value to three decimal places.

16. $\sqrt{129}$ 17. $-\sqrt{147}$ 18. $\sqrt{0.87}$

19. $\sqrt{4.27}$ 20. $\sqrt[3]{59}$ 21. $\sqrt[3]{-480}$

22. $\sqrt[4]{602}$ 23. $\sqrt[5]{891}$ 24. $\sqrt[6]{4123}$

25. $\sqrt[7]{46{,}815}$ 26. $\sqrt[6]{(723)^3}$ 27. $\sqrt[4]{(3500)^2}$

Simplify.

28. $\sqrt{225}$ 29. $\pm\sqrt{169}$ 30. $\sqrt{-(-7)^2}$

31. $\sqrt{(-18)^2}$ 32. $\sqrt[3]{-27}$ 33. $\sqrt[7]{-128}$

34. $\sqrt{\frac{1}{16}}$ 35. $\sqrt[3]{\frac{1}{125}}$ 36. $\sqrt{0.25}$

37. $\sqrt[3]{-0.064}$ 38. $\sqrt[4]{z^8}$ 39. $-\sqrt[6]{x^6}$

40. $\sqrt{49m^6}$ 41. $\sqrt{64a^8}$ 42. $\sqrt[3]{27r^3}$

43. $\sqrt[3]{-c^6}$ 44. $\sqrt{(5g)^4}$ 45. $\sqrt[3]{(2z)^6}$

46. $\sqrt{25x^4y^6}$ 47. $\sqrt{36x^4z^4}$ 48. $\sqrt{169x^8y^4}$

49. $\sqrt{9p^{12}q^6}$ 50. $\sqrt[3]{8a^3b^3}$ 51. $\sqrt[3]{-27c^9d^{12}}$

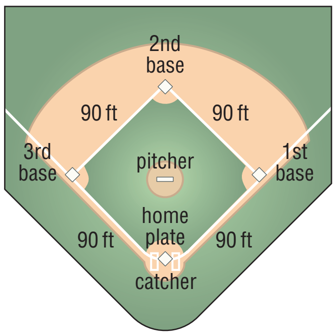
52. $\sqrt{(4x - y)^2}$ 53. $\sqrt[3]{(p + q)^3}$ 54. $-\sqrt{x^2 + 4x + 4}$

55. $\sqrt{z^2 + 8z + 16}$ 56. $\sqrt{4a^2 + 4a + 1}$ 57. $\sqrt{-9x^2 - 12x - 4}$

58. Find the principal fifth root of 32.

59. What is the third root of -125?

60. **SPORTS** Refer to the drawing at the right. How far does the catcher have to throw a ball from home plate to second base?



61. **FISH** The relationship between the length and mass of Pacific halibut can be approximated by the equation $L = 0.46\sqrt[3]{M}$, where L is the length in meters and M is the mass in kilograms. Use this equation to predict the length of a 25-kilogram Pacific halibut.

More About. . .

Space Science

The escape velocity for the Moon is about 2400 m/s. For the Sun, it is about 618,000 m/s.

Source: NASA

62. SPACE SCIENCE The velocity v required for an object to escape the gravity of a planet or other body is given by the formula $v = \sqrt{\frac{2GM}{R}}$, where M is the mass of the body, R is the radius of the body, and G is Newton's gravitational constant. Use $M = 5.98 \times 10^{24}$ kg, $R = 6.37 \times 10^6$ m, and $G = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$ to find the escape velocity for Earth.

63. CRITICAL THINKING Under what conditions does $\sqrt{x^2 + y^2} = x + y$?

64. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How do square roots apply to oceanography?

Include the following in your answer:

- the values of s for $\ell = 2, 5,$ and 10 feet, and
- an observation of what happens to the value of s as the value of ℓ increases.

Standardized Test Practice

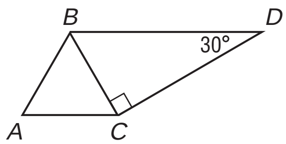
65. Which of the following is closest to $\sqrt{7.32}$?

Ⓐ 2.6 Ⓑ 2.7 Ⓒ 2.8 Ⓓ 2.9

66. In the figure, $\triangle ABC$ is an equilateral triangle with sides 9 units long. What is the length of $\overline{BD}$ in units?

Ⓐ 3 Ⓑ 9

Ⓒ $9\sqrt{2}$ Ⓓ 18



Maintain Your Skills

Mixed Review

Factor completely. If the polynomial is not factorable, write *prime*. *(Lesson 5-4)*

67. $7xy^3 - 14x^2y^5 + 28x^3y^2$

68. $ab - 5a + 3b - 15$

69. $2x^2 + 15x + 25$

70. $c^3 - 216$

Simplify. *(Lesson 5-3)*

71. $(4x^3 - 7x^2 + 3x - 2) \div (x - 2)$

72. $\dfrac{x^4 + 4x^3 - 4x^2 + 5x}{x + 5}$

73. TRAVEL The matrix at the right shows the costs of airline flights between some cities. Write a matrix that shows the costs of two tickets for these flights. *(Lesson 4-2)*

	New York	LA
Atlanta	405	1160
Chicago	709	1252

Solve each system of equations by using either substitution or elimination. *(Lesson 3-2)*

74. $a + 4b = 6$
$3a + 2b = -2$

75. $10x - y = 13$
$3x - 4y = 15$

76. $3c - 7d = -1$
$2c - 6d = -6$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each product.**
*(To review **multiplying binomials**, see Lesson 5-2.)*

77. $(x + 3)(x + 8)$

78. $(y - 2)(y + 5)$

79. $(a + 2)(a - 9)$

80. $(a + b)(a + 2b)$

81. $(x - 3y)(x + 3y)$

82. $(2w + z)(3w - 5z)$

5-6 Radical Expressions

What You'll Learn

- Simplify radical expressions.
- Add, subtract, multiply, and divide radical expressions.

How do radical expressions apply to falling objects?

The amount of time t in seconds that it takes for an object to drop d feet is given by $t = \sqrt{\frac{2d}{g}}$, where $g = 32$ ft/s^2 is the acceleration due to gravity. In this lesson, you will learn how to simplify radical expressions like $\sqrt{\frac{2d}{g}}$.

Vocabulary

- rationalizing the denominator
- like radical expressions
- conjugates

SIMPLIFY RADICAL EXPRESSIONS You can use the Commutative Property and the definition of square root to find an equivalent expression for a product of radicals such as $\sqrt{3} \cdot \sqrt{5}$. Begin by squaring the product.

$$\left(\sqrt{3} \cdot \sqrt{5}\right)^2 = \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{3} \cdot \sqrt{5}$$

$$= \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{5} \quad \text{Commutative Property of Multiplication}$$

$$= 3 \cdot 5 \text{ or } 15 \quad \text{Definition of square root}$$

Since $\sqrt{3} \cdot \sqrt{5} > 0$ and $\left(\sqrt{3} \cdot \sqrt{5}\right)^2 = 15$, $\sqrt{3} \cdot \sqrt{5}$ is the principal square root of 15. That is, $\sqrt{3} \cdot \sqrt{5} = \sqrt{15}$. This illustrates the following property of radicals.

Key Concept — Product Property of Radicals

For any real numbers a and b and any integer $n > 1$,

1. if n is even and a and b are both nonnegative, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, and

2. if n is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

Follow these steps to simplify a square root.

Step 1 Factor the radicand into as many squares as possible.

Step 2 Use the Product Property to isolate the perfect squares.

Step 3 Simplify each radical.

Example 1 Square Root of a Product

Simplify $\sqrt{16p^8q^7}$.

$$\sqrt{16p^8q^7} = \sqrt{4^2 \cdot (p^4)^2 \cdot (q^3)^2 \cdot q} \quad \text{Factor into squares where possible.}$$

$$= \sqrt{4^2} \cdot \sqrt{(p^4)^2} \cdot \sqrt{(q^3)^2} \cdot \sqrt{q} \quad \text{Product Property of Radicals}$$

$$= 4p^4\left|q^3\right|\sqrt{q} \quad \text{Simplify.}$$

However, for $\sqrt{16p^8q^7}$ to be defined, $16p^8q^7$ must be nonnegative. If that is true, q must be nonnegative, since it is raised to an odd power. Thus, the absolute value is unnecessary, and $\sqrt{16p^8q^7} = 4p^4q^3\sqrt{q}$.

Look at a radical that involves division to see if there is a quotient property for radicals that is similar to the Product Property. Consider $\frac{49}{9}$. The radicand is a perfect square, so $\sqrt{\frac{49}{9}} = \sqrt{\left(\frac{7}{3}\right)^2}$ or $\frac{7}{3}$. Notice that $\frac{7}{3} = \frac{\sqrt{49}}{\sqrt{9}}$. This suggests the following property.

Key Concept — Quotient Property of Radicals

- **Words** For any real numbers a and $b \neq 0$, and any integer $n > 1$, $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined.
- **Example** $\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{9}$ or 3

You can use the properties of radicals to write expressions in simplified form.

Concept Summary — Simplifying Radical Expressions

A radical expression is in simplified form when the following conditions are met.

- The index n is as small as possible.
- The radicand contains no factors (other than 1) that are nth powers of an integer or polynomial.
- The radicand contains no fractions.
- No radicals appear in a denominator.

Study Tip

Rationalizing the Denominator
You may want to think of rationalizing the denominator as making the denominator a rational number.

To eliminate radicals from a denominator or fractions from a radicand, you can use a process called **rationalizing the denominator**. To rationalize a denominator, multiply the numerator and denominator by a quantity so that the radicand has an exact root. Study the examples below.

Example 2 Simplify Quotients

Simplify each expression.

a. $\sqrt{\frac{x^4}{y^5}}$

$\sqrt{\frac{x^4}{y^5}} = \frac{\sqrt{x^4}}{\sqrt{y^5}}$ Quotient Property

$= \frac{\sqrt{(x^2)^2}}{\sqrt{(y^2)^2 \cdot y}}$ Factor into squares.

$= \frac{\sqrt{(x^2)^2}}{\sqrt{(y^2)^2} \cdot \sqrt{y}}$ Product Property

$= \frac{x^2}{y^2\sqrt{y}}$ $\sqrt{(x^2)^2} = x^2$

$= \frac{x^2}{y^2\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}}$ Rationalize the denominator.

$= \frac{x^2\sqrt{y}}{y^3}$ $\sqrt{y} \cdot \sqrt{y} = y$

b. $\sqrt[5]{\frac{5}{4a}}$

$\sqrt[5]{\frac{5}{4a}} = \frac{\sqrt[5]{5}}{\sqrt[5]{4a}}$ Quotient Property

$= \frac{\sqrt[5]{5}}{\sqrt[5]{4a}} \cdot \frac{\sqrt[5]{8a^4}}{\sqrt[5]{8a^4}}$ Rationalize the denominator.

$= \frac{\sqrt[5]{5 \cdot 8a^4}}{\sqrt[5]{4a \cdot 8a^4}}$ Product Property

$= \frac{\sqrt[5]{40a^4}}{\sqrt[5]{32a^5}}$ Multiply.

$= \frac{\sqrt[5]{40a^4}}{2a}$ $\sqrt[5]{32a^5} = 2a$

www.algebra2.com/extra_examples

OPERATIONS WITH RADICALS

You can use the Product and Quotient Properties to multiply and divide some radicals, respectively.

Example 3 Multiply Radicals

Simplify $6\sqrt[3]{9n^2} \cdot 3\sqrt[3]{24n}$.

$6\sqrt[3]{9n^2} \cdot 3\sqrt[3]{24n} = 6 \cdot 3 \cdot \sqrt[3]{9n^2 \cdot 24n}$ Product Property of Radicals

$= 18 \cdot \sqrt[3]{2^3 \cdot 3^3 \cdot n^3}$ Factor into cubes where possible.

$= 18 \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{3^3} \cdot \sqrt[3]{n^3}$ Product Property of Radicals

$= 18 \cdot 2 \cdot 3 \cdot n$ or $108n$ Multiply.

Can you add radicals in the same way that you multiply them? In other words, if $\sqrt{a} \cdot \sqrt{a} = \sqrt{a \cdot a}$, does $\sqrt{a} + \sqrt{a} = \sqrt{a + a}$?

Algebra Activity

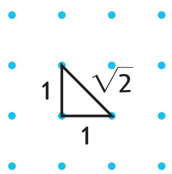
Adding Radicals

You can use dot paper to show the sum of two like radicals, such as $\sqrt{2} + \sqrt{2}$.

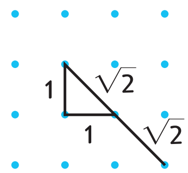
Model and Analyze

Step 1 First, find a segment of length $\sqrt{2}$ units by using the Pythagorean Theorem with the dot paper.

$a^2 + b^2 = c^2$
$1^2 + 1^2 = c^2$
$2 = c^2$



Step 2 Extend the segment to twice its length to represent $\sqrt{2} + \sqrt{2}$.



Make a Conjecture

1. Is $\sqrt{2} + \sqrt{2} = \sqrt{2 + 2}$ or 2? Justify your answer using the geometric models above.
2. Use this method to model other irrational numbers. Do these models support your conjecture?

In the activity, you discovered that you cannot add radicals in the same manner as you multiply them. You add radicals in the same manner as adding monomials. That is, you can add only the like terms or like radicals.

Study Tip

Reading Math
Indices is the plural of *index*.

Two radical expressions are called **like radical expressions** if both the indices and the radicands are alike. Some examples of like and unlike radical expressions are given below.

$\sqrt{3}$ and $\sqrt[3]{3}$ are not like expressions. Different indices

$\sqrt[4]{5x}$ and $\sqrt[4]{5}$ are not like expressions. Different radicands

$2\sqrt[4]{3a}$ and $5\sqrt[4]{3a}$ are like expressions. Radicands are $3a$; indices are 4.

Example 4 Add and Subtract Radicals

Simplify $2\sqrt{12} - 3\sqrt{27} + 2\sqrt{48}$.

$2\sqrt{12} - 3\sqrt{27} + 2\sqrt{48}$

$= 2\sqrt{2^2 \cdot 3} - 3\sqrt{3^2 \cdot 3} + 2\sqrt{2^2 \cdot 2^2 \cdot 3}$ — Factor using squares.

$= 2\sqrt{2^2} \cdot \sqrt{3} - 3\sqrt{3^2} \cdot \sqrt{3} + 2\sqrt{2^2} \cdot \sqrt{2^2} \cdot \sqrt{3}$ — Product Property

$= 2 \cdot 2 \cdot \sqrt{3} - 3 \cdot 3 \cdot \sqrt{3} + 2 \cdot 2 \cdot 2 \cdot \sqrt{3}$ — $\sqrt{2^2} = 2$, $\sqrt{3^2} = 3$

$= 4\sqrt{3} - 9\sqrt{3} + 8\sqrt{3}$ — Multiply.

$= 3\sqrt{3}$ — Combine like radicals.

Just as you can add and subtract radicals like monomials, you can multiply radicals using the FOIL method as you do when multiplying binomials.

Example 5 Multiply Radicals

Simplify each expression.

a. $(3\sqrt{5} - 2\sqrt{3})(2 + \sqrt{3})$

$(3\sqrt{5} - 2\sqrt{3})(2 + \sqrt{3}) = \overset{F}{3\sqrt{5} \cdot 2} + \overset{O}{3\sqrt{5} \cdot \sqrt{3}} - \overset{I}{2\sqrt{3} \cdot 2} - \overset{L}{2\sqrt{3} \cdot \sqrt{3}}$

$= 6\sqrt{5} + 3\sqrt{5 \cdot 3} - 4\sqrt{3} - 2\sqrt{3^2}$ — Product Property

$= 6\sqrt{5} + 3\sqrt{15} - 4\sqrt{3} - 6$ — $2\sqrt{3^2} = 2 \cdot 3$ or 6

b. $(5\sqrt{3} - 6)(5\sqrt{3} + 6)$

$(5\sqrt{3} - 6)(5\sqrt{3} + 6) = 5\sqrt{3} \cdot 5\sqrt{3} + 5\sqrt{3} \cdot 6 - 6 \cdot 5\sqrt{3} - 6 \cdot 6$ — FOIL

$= 25\sqrt{3^2} + 30\sqrt{3} - 30\sqrt{3} - 36$ — Multiply.

$= 75 - 36$ — $25\sqrt{3^2} = 25 \cdot 3$ or 75

$= 39$ — Subtract.

Binomials like those in Example 5b, of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ where a, b, c, and d are rational numbers, are called **conjugates** of each other. The product of conjugates is always a rational number. You can use conjugates to rationalize denominators.

Example 6 Use a Conjugate to Rationalize a Denominator

Simplify $\dfrac{1 - \sqrt{3}}{5 + \sqrt{3}}$.

$\dfrac{1 - \sqrt{3}}{5 + \sqrt{3}} = \dfrac{(1 - \sqrt{3})(5 - \sqrt{3})}{(5 + \sqrt{3})(5 - \sqrt{3})}$ — Multiply by $\dfrac{5 - \sqrt{3}}{5 - \sqrt{3}}$ because $5 - \sqrt{3}$ is the conjugate of $5 + \sqrt{3}$.

$= \dfrac{1 \cdot 5 - 1 \cdot \sqrt{3} - \sqrt{3} \cdot 5 + (\sqrt{3})^2}{5^2 - (\sqrt{3})^2}$ — FOIL; Difference of squares

$= \dfrac{5 - \sqrt{3} - 5\sqrt{3} + 3}{25 - 3}$ — Multiply.

$= \dfrac{8 - 6\sqrt{3}}{22}$ — Combine like terms.

$= \dfrac{4 - 3\sqrt{3}}{11}$ — Divide numerator and denominator by 2.

Check for Understanding

Concept Check

1. **Determine** whether the statement $\frac{1}{\sqrt[n]{a}} = \sqrt[n]{a}$ is *sometimes*, *always*, or *never* true. Explain.
2. **OPEN ENDED** **Write** a sum of three radicals that contains two like terms.
3. **Explain** why the product of two conjugates is always a rational number.

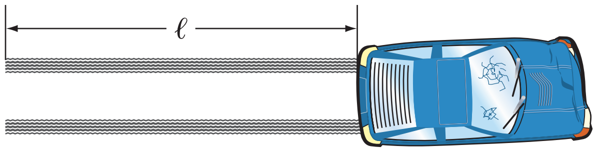
Guided Practice

Simplify.

4. $5\sqrt{63}$
5. $\sqrt[4]{16x^5y^4}$
6. $\sqrt{\frac{7}{8y}}$
7. $(-2\sqrt{15})(4\sqrt{21})$
8. $\frac{\sqrt[3]{625}}{\sqrt[3]{25}}$
9. $\sqrt{2ab^2} \cdot \sqrt{6a^3b^2}$
10. $\sqrt{3} - 2\sqrt[4]{3} + 4\sqrt{3} + 5\sqrt[4]{3}$
11. $3\sqrt[3]{128} + 5\sqrt[3]{16}$
12. $(3 - \sqrt{5})(1 + \sqrt{3})$
13. $\frac{1 + \sqrt{5}}{3 - \sqrt{5}}$

Application

14. **LAW ENFORCEMENT** A police accident investigator can use the formula $s = 2\sqrt{5\ell}$ to estimate the speed s of a car in miles per hour based on the length ℓ in feet of the skid marks it left. How fast was a car traveling that left skid marks 120 feet long?



Practice and Apply

Homework Help

For Exercises	See Examples
15–26	1
27–30	2
31–34	3
35–38	4
39–42	5
43–48	6

Extra Practice
See page 838.

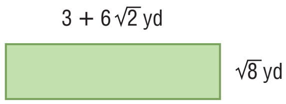
Simplify.

15. $\sqrt{243}$
16. $\sqrt{72}$
17. $\sqrt[3]{54}$
18. $\sqrt[4]{96}$
19. $\sqrt{50x^4}$
20. $\sqrt[3]{16y^3}$
21. $\sqrt{18x^2y^3}$
22. $\sqrt{40a^3b^4}$
23. $3\sqrt[3]{56y^6z^3}$
24. $2\sqrt[3]{24m^4n^5}$
25. $\sqrt[4]{\frac{1}{81}c^5d^4}$
26. $\sqrt[5]{\frac{1}{32}w^6z^7}$
27. $\sqrt[3]{\frac{3}{4}}$
28. $\sqrt[4]{\frac{2}{3}}$
29. $\sqrt{\frac{a^4}{b^3}}$
30. $\sqrt{\frac{4r^8}{t^9}}$
31. $(3\sqrt{12})(2\sqrt{21})$
32. $(-3\sqrt{24})(5\sqrt{20})$
33. What is $\sqrt{39}$ divided by $\sqrt{26}$?
34. Divide $\sqrt{14}$ by $\sqrt{35}$.

Simplify.

35. $\sqrt{12} + \sqrt{48} - \sqrt{27}$
36. $\sqrt{98} - \sqrt{72} + \sqrt{32}$
37. $\sqrt{3} + \sqrt{72} - \sqrt{128} + \sqrt{108}$
38. $5\sqrt{20} + \sqrt{24} - \sqrt{180} + 7\sqrt{54}$
39. $(5 + \sqrt{6})(5 - \sqrt{2})$
40. $(3 + \sqrt{7})(2 + \sqrt{6})$
41. $(\sqrt{11} - \sqrt{2})^2$
42. $(\sqrt{3} - \sqrt{5})^2$
43. $\frac{7}{4 - \sqrt{3}}$
44. $\frac{\sqrt{6}}{5 + \sqrt{3}}$
45. $\frac{-2 - \sqrt{3}}{1 + \sqrt{3}}$
46. $\frac{2 + \sqrt{2}}{5 - \sqrt{2}}$
47. $\frac{x + 1}{\sqrt{x^2 - 1}}$
48. $\frac{x - 1}{\sqrt{x - 1}}$

49. **GEOMETRY** Find the perimeter and area of the rectangle.



More About. . .

Amusement Parks

Attendance at the top 50 theme parks in North America increased to 175.1 million in 2000.

Source: *Amusement Business*

AMUSEMENT PARKS **For Exercises 50 and 51, use the following information.**
The velocity v in feet per second of a roller coaster at the bottom of a hill is related to the vertical drop h in feet and the velocity v_0 in feet per second of the coaster at the top of the hill by the formula $v_0 = \sqrt{v^2 - 64h}$.

50. Explain why $v_0 = v - 8\sqrt{h}$ is not equivalent to the given formula.

51. What velocity must a coaster have at the top of a 225-foot hill to achieve a velocity of 120 feet per second at the bottom?

Online Research **Data Update** What are the values of v and h for some of the world's highest and fastest roller coasters? Visit www.algebra2.com/data_update to learn more.

SPORTS **For Exercises 52 and 53, use the following information.**
A ball that is hit or thrown horizontally with a velocity of v meters per second will travel a distance of d meters before hitting the ground, where $d = v\sqrt{\frac{h}{4.9}}$ and h is the height in meters from which the ball is hit or thrown.

52. Use the properties of radicals to rewrite the formula.

53. How far will a ball that is hit horizontally with a velocity of 45 meters per second at a height of 0.8 meter above the ground travel before hitting the ground?

54. **AUTOMOTIVE ENGINEERING** An automotive engineer is trying to design a safer car. The maximum force a road can exert on the tires of the car being redesigned is 2000 pounds. What is the maximum velocity v in ft/s at which this car can safely round a turn of radius 320 feet? Use the formula $v = \sqrt{\frac{F_c r}{100}}$, where F_c is the force the road exerts on the car and r is the radius of the turn.

55. **CRITICAL THINKING** Under what conditions is the equation $\sqrt{x^3y^2} = xy\sqrt{x}$ true?

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do radical expressions apply to falling objects?

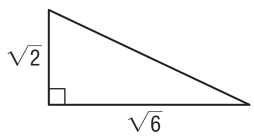
Include the following in your answer:
- an explanation of how you can use the properties in this lesson to rewrite the formula $t = \sqrt{\frac{2d}{g}}$, and
- the amount of time a 5-foot tall student has to get out of the way after a balloon is dropped from a window 25 feet above.

Standardized Test Practice

57. The expression $\sqrt{180}$ is equivalent to which of the following?

Ⓐ $5\sqrt{6}$ Ⓑ $6\sqrt{5}$ Ⓒ $3\sqrt{10}$ Ⓓ $36\sqrt{5}$

58. Which of the following is *not* a length of a side of the triangle?



Ⓐ $\sqrt{8}$ Ⓑ $2\sqrt{2}$
Ⓒ $\sqrt{4 + 2}$ Ⓓ $\sqrt{4} + \sqrt{2}$

Maintain Your Skills

Mixed Review

Simplify. *(Lesson 5-5)*

59. $\sqrt{144z^8}$ **60.** $\sqrt[3]{216a^3b^9}$ **61.** $\sqrt{(y+2)^2}$

Simplify. Assume that no denominator is equal to 0. *(Lesson 5-4)*

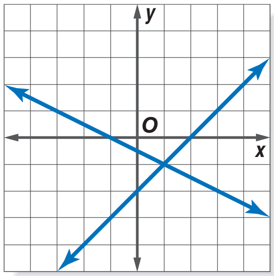
62. $\dfrac{x^2+5x-14}{x^2-6x+8}$ **63.** $\dfrac{x^2-3x-4}{x^2-16}$

Perform the indicated operations. *(Lesson 4-2)*

64. $\begin{bmatrix} 3 & -4 \\ 2 & 8 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -5 & 0 \\ 7 & 7 \\ 3 & -6 \end{bmatrix}$ **65.** $\begin{bmatrix} 3 & 3 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 5 & 2 \end{bmatrix}$

66. Find the maximum and minimum values of the function $f(x, y) = 2x + 3y$ for the region with vertices at (2, 4), (−1, 3), (−3, −3), and (2, −5). *(Lesson 3-4)*

67. State whether the system of equations shown at the right is *consistent and independent*, *consistent and dependent*, or *inconsistent*. *(Lesson 3-1)*



68. BUSINESS The amount that a mail-order company charges for shipping and handling is given by the function $c(x) = 3 + 0.15x$, where x is the weight in pounds. Find the charge for an 8-pound order. *(Lesson 2-2)*

Solve. *(Lessons 1-3, 1-4, and 1-5)*

69. $2x + 7 = -3$ **70.** $-5x + 6 = -4$

71. $|x - 1| = 3$ **72.** $|3x + 2| = 5$

73. $2x - 4 > 8$ **74.** $-x - 3 \le 4$

Getting Ready for the Next Lesson

BASIC SKILL **Evaluate each expression.**

75. $2\left(\frac{1}{8}\right)$ **76.** $3\left(\frac{1}{6}\right)$ **77.** $\frac{1}{2} + \frac{1}{3}$ **78.** $\frac{1}{3} + \frac{3}{4}$

79. $\frac{1}{8} + \frac{5}{12}$ **80.** $\frac{5}{6} - \frac{1}{5}$ **81.** $\frac{5}{8} - \frac{1}{4}$ **82.** $\frac{1}{4} - \frac{2}{3}$

Practice Quiz 2 — *Lessons 5-4 through 5-6*

Factor completely. If the polynomial is not factorable, write prime. *(Lesson 5-4)*

1. $3x^3y + x^2y^2 + x^2y$ **2.** $3x^2 - 2x - 2$

3. $ax^2 + 6ax + 9a$ **4.** $8r^3 - 64s^6$

Simplify. *(Lessons 5-5 and 5-6)*

5. $\sqrt{36x^2y^6}$ **6.** $\sqrt[3]{-64a^6b^9}$ **7.** $\sqrt{4n^2 + 12n + 9}$

8. $\sqrt{\dfrac{x^4}{y^3}}$ **9.** $(3 + \sqrt{7})(2 - \sqrt{7})$ **10.** $\dfrac{5 + \sqrt{2}}{2 + \sqrt{2}}$

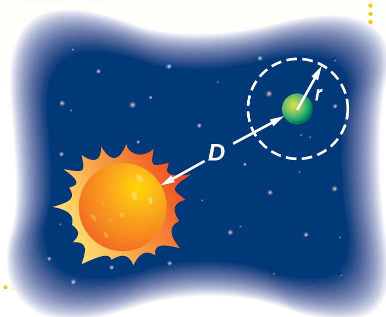
5-7 Rational Exponents

What You'll Learn

- Write expressions with rational exponents in radical form, and vice versa.
- Simplify expressions in exponential or radical form.

How do rational exponents apply to astronomy?

Astronomers refer to the space around a planet where the planet's gravity is stronger than the Sun's as the *sphere of influence* of the planet. The radius r of the sphere of influence is given by the formula $r = D\left(\frac{M_p}{M_S}\right)^{\frac{2}{5}}$, where M_p is the mass of the planet, M_S is the mass of the Sun, and D is the distance between the planet and the Sun.



RATIONAL EXPONENTS AND RADICALS You know that squaring a number and taking the square root of a number are inverse operations. But how would you evaluate an expression that contains a fractional exponent such as the one above? You can investigate such an expression by assuming that fractional exponents behave as integral exponents.

$$\left(b^{\frac{1}{2}}\right)^2 = b^{\frac{1}{2}} \cdot b^{\frac{1}{2}} \quad \text{Write the square as multiplication.}$$
$$= b^{\frac{1}{2} + \frac{1}{2}} \quad \text{Add the exponents.}$$
$$= b^1 \text{ or } b \quad \text{Simplify.}$$

Thus, $b^{\frac{1}{2}}$ is a number whose square equals b. So it makes sense to define $b^{\frac{1}{2}} = \sqrt{b}$.

Key Concept $b^{\frac{1}{n}}$

- **Words** For any real number b and for any positive integer n, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and n is even.
- **Example** $8^{\frac{1}{3}} = \sqrt[3]{8}$ or 2

Example 1 Radical Form

Write each expression in radical form.

a. $a^{\frac{1}{4}}$

$a^{\frac{1}{4}} = \sqrt[4]{a}$ Definition of $b^{\frac{1}{n}}$

b. $x^{\frac{1}{5}}$

$x^{\frac{1}{5}} = \sqrt[5]{x}$ Definition of $b^{\frac{1}{n}}$

Example 2 Exponential Form

Write each radical using rational exponents.

a. $\sqrt[3]{y}$

$\sqrt[3]{y} = y^{\frac{1}{3}}$ Definition of $b^{\frac{1}{n}}$

b. $\sqrt[8]{c}$

$\sqrt[8]{c} = c^{\frac{1}{8}}$ Definition of $b^{\frac{1}{n}}$

Many expressions with fractional exponents can be evaluated using the definition of $b^{\frac{1}{n}}$ or the properties of powers.

Study Tip

Negative Base
Suppose the base of a monomial is negative such as $(-9)^2$ or $(-9)^3$. The expression is undefined if the exponent is even because there is no number that, when multiplied an even number of times, results in a negative number. However, the expression is defined for an odd exponent.

Example 3 Evaluate Expressions with Rational Exponents

Evaluate each expression.

a. $16^{-\frac{1}{4}}$

Method 1

$16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}}$ $b^{-n} = \frac{1}{b^n}$

$= \frac{1}{\sqrt[4]{16}}$ $16^{\frac{1}{4}} = \sqrt[4]{16}$

$= \frac{1}{\sqrt[4]{2^4}}$ $16 = 2^4$

$= \frac{1}{2}$ Simplify.

Method 2

$16^{-\frac{1}{4}} = (2^4)^{-\frac{1}{4}}$ $16 = 2^4$

$= 2^{4\left(-\frac{1}{4}\right)}$ Power of a Power

$= 2^{-1}$ Multiply exponents.

$= \frac{1}{2}$ $2^{-1} = \frac{1}{2^1}$

b. $243^{\frac{3}{5}}$

Method 1

$243^{\frac{3}{5}} = 243^{3\left(\frac{1}{5}\right)}$ Factor.

$= (243^3)^{\frac{1}{5}}$ Power of a Power

$= \sqrt[5]{243^3}$ $b^{\frac{1}{5}} = \sqrt[5]{b}$

$= \sqrt[5]{(3^5)^3}$ $243 = 3^5$

$= \sqrt[5]{3^5 \cdot 3^5 \cdot 3^5}$ Expand the cube.

$= 3 \cdot 3 \cdot 3$ or 27 Find the fifth root.

Method 2

$243^{\frac{3}{5}} = (3^5)^{\frac{3}{5}}$ $243 = 3^5$

$= 3^{5\left(\frac{3}{5}\right)}$ Power of a Power

$= 3^3$ Multiply exponents.

$= 27$ $3^3 = 3 \cdot 3 \cdot 3$

In Example 3b, Method 1 uses a combination of the definition of $b^{\frac{1}{n}}$ and the properties of powers. This example suggests the following general definition of rational exponents.

Key Concept — Rational Exponents

- **Words** For any nonzero real number b, and any integers m and n, with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$, except when $b < 0$ and n is even.
- **Example** $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2$ or 4

In general, we define $b^{\frac{m}{n}}$ as $\left(b^{\frac{1}{n}}\right)^m$ or $(b^m)^{\frac{1}{n}}$. Now apply the definition of $b^{\frac{1}{n}}$ to $\left(b^{\frac{1}{n}}\right)^m$ and $(b^m)^{\frac{1}{n}}$.

$\left(b^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{b}\right)^m$ $(b^m)^{\frac{1}{n}} = \sqrt[n]{b^m}$

Example 4 Rational Exponent with Numerator Other Than 1

WEIGHT LIFTING **The formula $M = 512 - 146{,}230B^{-\frac{8}{5}}$ can be used to estimate the maximum total mass that a weight lifter of mass B kilograms can lift in two lifts, the snatch and the clean and jerk, combined.**

a. According to the formula, what is the maximum amount that 2000 Olympic champion Xugang Zhan of China can lift if he weighs 72 kilograms?

$M = 512 - 146{,}230B^{-\frac{8}{5}}$ Original formula

$= 512 - 146{,}230(72)^{-\frac{8}{5}}$ $B = 72$

≈ 356 kg Use a calculator.

The formula predicts that he can lift at most 356 kilograms.

b. Xugang Zhan's winning total in the 2000 Olympics was 367.50 kg. Compare this to the value predicted by the formula.

The formula prediction is close to the actual weight, but slightly lower.

More About...

Weight Lifting

With origins in both the ancient Egyptian and Greek societies, weightlifting was among the sports on the program of the first Modern Olympic Games, in 1896, in Athens, Greece.

Source: International Weightlifting Association

SIMPLIFY EXPRESSIONS All of the properties of powers you learned in Lesson 5-1 apply to rational exponents. When simplifying expressions containing rational exponents, leave the exponent in rational form rather than writing the expression as a radical. To simplify such an expression, you must write the expression with all positive exponents. Furthermore, any exponents in the denominator of a fraction must be positive *integers*. So, it may be necessary to rationalize a denominator.

Example 5 Simplify Expressions with Rational Exponents

Simplify each expression.

a. $x^{\frac{1}{5}} \cdot x^{\frac{7}{5}}$

$x^{\frac{1}{5}} \cdot x^{\frac{7}{5}} = x^{\frac{1}{5} + \frac{7}{5}}$ Multiply powers.

$= x^{\frac{8}{5}}$ Add exponents.

b. $y^{-\frac{3}{4}}$

$y^{-\frac{3}{4}} = \frac{1}{y^{\frac{3}{4}}}$ $b^{-n} = \frac{1}{b^n}$

$= \frac{1}{y^{\frac{3}{4}}} \cdot \frac{y^{\frac{1}{4}}}{y^{\frac{1}{4}}}$ Why use $\frac{y^{\frac{1}{4}}}{y^{\frac{1}{4}}}$?

$= \frac{y^{\frac{1}{4}}}{y^{\frac{4}{4}}}$ $y^{\frac{3}{4}} \cdot y^{\frac{1}{4}} = y^{\frac{3}{4} + \frac{1}{4}}$

$= \frac{y^{\frac{1}{4}}}{y}$ $y^{\frac{4}{4}} = y^1$ or y

When simplifying a radical expression, always use the smallest index possible. Using rational exponents makes this process easier, but the answer should be written in radical form.

Example 6 Simplify Radical Expressions

Simplify each expression.

a. $\dfrac{\sqrt[8]{81}}{\sqrt[6]{3}}$

$\dfrac{\sqrt[8]{81}}{\sqrt[6]{3}} = \dfrac{81^{\frac{1}{8}}}{3^{\frac{1}{6}}}$ Rational exponents

$= \dfrac{(3^4)^{\frac{1}{8}}}{3^{\frac{1}{6}}}$ $81 = 3^4$

$= \dfrac{3^{\frac{1}{2}}}{3^{\frac{1}{6}}}$ Power of a Power

$= 3^{\frac{1}{2} - \frac{1}{6}}$ Quotient of Powers

$= 3^{\frac{1}{3}}$ or $\sqrt[3]{3}$ Simplify.

b. $\sqrt[4]{9z^2}$

$\sqrt[4]{9z^2} = (9z^2)^{\frac{1}{4}}$ Rational exponents

$= (3^2 \cdot z^2)^{\frac{1}{4}}$ $9 = 3^2$

$= 3^{2\left(\frac{1}{4}\right)} \cdot z^{2\left(\frac{1}{4}\right)}$ Power of a Power

$= 3^{\frac{1}{2}} \cdot z^{\frac{1}{2}}$ Multiply.

$= \sqrt{3} \cdot \sqrt{z}$ $3^{\frac{1}{2}} = \sqrt{3}, z^{\frac{1}{2}} = \sqrt{z}$

$= \sqrt{3z}$ Simplify.

c. $\dfrac{m^{\frac{1}{2}} - 1}{m^{\frac{1}{2}} + 1}$

$\dfrac{m^{\frac{1}{2}} - 1}{m^{\frac{1}{2}} + 1} = \dfrac{m^{\frac{1}{2}} - 1}{m^{\frac{1}{2}} + 1} \cdot \dfrac{m^{\frac{1}{2}} - 1}{m^{\frac{1}{2}} - 1}$ $m^{\frac{1}{2}} - 1$ is the conjugate of $m^{\frac{1}{2}} + 1$.

$= \dfrac{m - 2m^{\frac{1}{2}} + 1}{m - 1}$ Multiply.

Concept Summary — Expressions with Rational Exponents

An expression with rational exponents is simplified when all of the following conditions are met.

- It has no negative exponents.
- It has no fractional exponents in the denominator.
- It is not a complex fraction.
- The index of any remaining radical is the least number possible.

Check for Understanding

Concept Check

1. **OPEN ENDED** Determine a value of b for which $b^{\frac{1}{6}}$ is an integer.
2. **Explain** why $(-16)^{\frac{1}{2}}$ is not a real number.
3. **Explain** why $\sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$.

Guided Practice

Write each expression in radical form.

4. $7^{\frac{1}{3}}$
5. $x^{\frac{2}{3}}$

Write each radical using rational exponents.

6. $\sqrt[4]{26}$
7. $\sqrt[3]{6x^5y^7}$

Evaluate each expression.

8. $125^{\frac{1}{3}}$
9. $81^{-\frac{1}{4}}$
10. $27^{\frac{2}{3}}$
11. $\dfrac{54}{9^{\frac{3}{2}}}$

Simplify each expression.

12. $a^{\frac{2}{3}} \cdot a^{\frac{1}{4}}$
13. $\dfrac{x^{\frac{5}{6}}}{x^{\frac{1}{6}}}$
14. $\dfrac{1}{2z^{\frac{1}{3}}}$
15. $\dfrac{a^2}{b^{\frac{1}{3}}} \cdot \dfrac{b}{a^{\frac{1}{2}}}$
16. $(mn^2)^{-\frac{1}{3}}$
17. $z(x - 2y)^{-\frac{1}{2}}$
18. $\sqrt[6]{27x^3}$
19. $\dfrac{\sqrt[4]{27}}{\sqrt[4]{3}}$

Application

20. **ECONOMICS** When inflation causes the price of an item to increase, the new cost C and the original cost c are related by the formula $C = c(1 + r)^n$, where r is the rate of inflation per year as a decimal and n is the number of years. What would be the price of a \$4.99 item after six months of 5% inflation?

Practice and Apply

Homework Help

For Exercises	See Examples
21–24	1
25–28	2
29–40	3
41–52, 64–66	5
53–63	6

Extra Practice
See page 838.

Write each expression in radical form.

21. $6^{\frac{1}{5}}$
22. $4^{\frac{1}{3}}$
23. $c^{\frac{2}{5}}$
24. $(x^2)^{\frac{4}{3}}$

Write each radical using rational exponents.

25. $\sqrt{23}$
26. $\sqrt[3]{62}$
27. $\sqrt[4]{16z^2}$
28. $\sqrt[3]{5x^2y}$

Evaluate each expression.

29. $16^{\frac{1}{4}}$
30. $216^{\frac{1}{3}}$
31. $25^{-\frac{1}{2}}$
32. $81^{-\frac{3}{4}}$
33. $(-27)^{-\frac{2}{3}}$
34. $(-32)^{-\frac{3}{5}}$
35. $81^{-\frac{1}{2}} \cdot 81^{\frac{3}{2}}$
36. $8^{\frac{3}{2}} \cdot 8^{\frac{5}{2}}$
37. $\left(\dfrac{8}{27}\right)^{\frac{1}{3}}$
38. $\left(\dfrac{1}{243}\right)^{-\frac{3}{5}}$
39. $\dfrac{16^{\frac{1}{2}}}{9^{\frac{1}{2}}}$
40. $\dfrac{8^{\frac{1}{3}}}{64^{\frac{1}{3}}}$

Simplify each expression.

41. $y^{\frac{5}{3}} \cdot y^{\frac{7}{3}}$
42. $x^{\frac{3}{4}} \cdot x^{\frac{9}{4}}$
43. $\left(b^{\frac{1}{3}}\right)^{\frac{3}{5}}$
44. $\left(a^{-\frac{2}{3}}\right)^{-\frac{1}{6}}$
45. $w^{-\frac{4}{5}}$
46. $x^{-\frac{1}{6}}$
47. $\dfrac{t^{\frac{3}{4}}}{t^{\frac{1}{2}}}$
48. $\dfrac{r^{\frac{2}{3}}}{r^{\frac{1}{6}}}$
49. $\dfrac{a^{-\frac{1}{2}}}{6a^{\frac{1}{3}} \cdot a^{-\frac{1}{4}}}$
50. $\dfrac{2c^{\frac{1}{8}}}{c^{-\frac{1}{16}} \cdot c^{\frac{1}{4}}}$
51. $\dfrac{y^{\frac{3}{2}}}{y^{\frac{1}{2}} + 2}$
52. $\dfrac{x^{\frac{1}{2}} + 2}{x^{\frac{1}{2}} - 1}$
53. $\sqrt[4]{25}$
54. $\sqrt[6]{27}$
55. $\sqrt{17} \cdot \sqrt[3]{17^2}$
56. $\sqrt[3]{5} \cdot \sqrt{5^3}$
57. $\sqrt[8]{25x^4y^4}$
58. $\sqrt[6]{81a^4b^8}$
59. $\dfrac{xy}{\sqrt{z}}$
60. $\dfrac{ab}{\sqrt[3]{c}}$
61. $\sqrt[3]{\sqrt{8}}$
62. $\sqrt{\sqrt[3]{36}}$
63. $\dfrac{8^{\frac{1}{6}} - 9^{\frac{1}{4}}}{\sqrt{3} + \sqrt{2}}$
64. $\dfrac{x^{\frac{5}{3}} - x^{\frac{1}{3}}z^{\frac{4}{3}}}{x^{\frac{2}{3}} + z^{\frac{2}{3}}}$

65. Find the simplified form of $32^{\frac{1}{2}} + 3^{\frac{1}{2}} - 8^{\frac{1}{2}}$.

66. What is the simplified form of $81^{\frac{1}{3}} - 24^{\frac{1}{3}} + 3^{\frac{1}{3}}$?

More About. . .

Music

The first piano was made in about 1709 by Bartolomeo Cristofori, a maker of harpsichords in Florence, Italy.

Source: www.infoplease.com

MUSIC For Exercises 67 and 68, use the following information.
On a piano, the frequency of the A note above middle C should be set at 440 vibrations per second. The frequency f_n of a note that is n notes above that A should be $f_n = 440 \cdot 2^{\frac{n}{12}}$.

67. At what frequency should a piano tuner set the A that is one octave, or 12 notes, above the A above middle C?

68. Middle C is nine notes below the A that has a frequency of 440 vibrations per second. What is the frequency of middle C?

69. **BIOLOGY** Suppose a culture has 100 bacteria to begin with and the number of bacteria doubles every 2 hours. Then the number N of bacteria after t hours is given by $N = 100 \cdot 2^{\frac{t}{2}}$. How many bacteria will be present after 3 and a half hours?

70. **CRITICAL THINKING** Explain how to solve $9^x = 3^{x + \frac{1}{2}}$ for x.

71. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do rational exponents apply to astronomy?

Include the following in your answer:

- an explanation of how to write the formula $r = D\left(\frac{M_p}{M_S}\right)^{\frac{2}{5}}$ in radical form and simplify it, and
- an explanation of what happens to the value of r as the value of D increases assuming that M_p and M_S are constant.

Standardized Test Practice

72. Which is the value of $4^{\frac{1}{2}} + \left(\frac{1}{2}\right)^4$?

Ⓐ 1 Ⓑ 2 Ⓒ $2\frac{1}{16}$ Ⓓ $2\frac{1}{2}$

73. If $4x + 2y = 5$ and $x - y = 1$, then what is the value of $3x + 3y$?

Ⓐ 1 Ⓑ 2 Ⓒ 4 Ⓓ 6

Maintain Your Skills

Mixed Review

Simplify. *(Lessons 5-5 and 5-6)*

74. $\sqrt{4x^3y^2}$

75. $(2\sqrt{6})(3\sqrt{12})$

76. $\sqrt{32} + \sqrt{18} - \sqrt{50}$

77. $\sqrt[4]{(-8)^4}$

78. $4\sqrt{(x - 5)^2}$

79. $\sqrt{\frac{9}{36}x^4}$

80. **BIOLOGY** Humans blink their eyes about once every 5 seconds. How many times do humans blink their eyes in two hours? *(Lesson 1-1)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each power. *(To review **multiplying radicals**, see Lesson 5-6.)*

81. $(\sqrt{x} - 2)^2$

82. $(\sqrt[3]{2x} - 3)^3$

83. $(\sqrt{x} + 1)^2$

84. $(2\sqrt{x} - 3)^2$

5-8 Radical Equations and Inequalities

What You'll Learn

- Solve equations containing radicals.
- Solve inequalities containing radicals.

Vocabulary

- radical equation
- extraneous solution
- radical inequality

How do radical equations apply to manufacturing?

Computer chips are made from the element silicon, which is found in sand. Suppose a company that manufactures computer chips uses the formula $C = 10n^{\frac{2}{3}} + 1500$ to estimate the cost C in dollars of producing n chips. This equation can be rewritten as a radical equation.

SOLVE RADICAL EQUATIONS Equations with radicals that have variables in the radicands are called **radical equations**. To solve this type of equation, raise each side of the equation to a power equal to the index of the radical to eliminate the radical.

Example 1 Solve a Radical Equation

Solve $\sqrt{x+1} + 2 = 4$.

$\sqrt{x+1} + 2 = 4$ Original equation

$\sqrt{x+1} = 2$ Subtract 2 from each side to isolate the radical.

$(\sqrt{x+1})^2 = 2^2$ Square each side to eliminate the radical.

$x + 1 = 4$ Find the squares.

$x = 3$ Subtract 1 from each side.

CHECK $\sqrt{x+1} + 2 = 4$ Original equation

$\sqrt{3+1} + 2 \stackrel{?}{=} 4$ Replace x with 3.

$4 = 4$ ✓ Simplify.

The solution checks. The solution is 3.

When you solve a radical equation, it is very important that you check your solution. Sometimes you will obtain a number that does not satisfy the original equation. Such a number is called an **extraneous solution**. You can use a graphing calculator to predict the number of solutions of an equation or to determine whether the solution you obtain is reasonable.

Example 2 Extraneous Solution

Solve $\sqrt{x-15} = 3 - \sqrt{x}$.

$\sqrt{x-15} = 3 - \sqrt{x}$ Original equation

$(\sqrt{x-15})^2 = (3 - \sqrt{x})^2$ Square each side.

$x - 15 = 9 - 6\sqrt{x} + x$ Find the squares.

$-24 = -6\sqrt{x}$ Isolate the radical.

$4 = \sqrt{x}$ Divide each side by -6.

$4^2 = (\sqrt{x})^2$ Square each side again.

$16 = x$ Evaluate the squares.

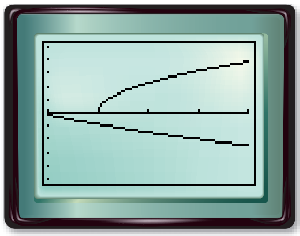
(continued on the next page)

CHECK

$$\sqrt{x - 15} = 3 - \sqrt{x}$$
$$\sqrt{16 - 15} \stackrel{?}{=} 3 - \sqrt{16}$$
$$\sqrt{1} \stackrel{?}{=} 3 - 4$$
$$1 \neq -1$$

The solution does not check, so the equation has no real solution.

The graphing calculator screen shows the graphs of $y = \sqrt{x - 15}$ and $y = 3 - \sqrt{x}$. The graphs do not intersect, which confirms that there is no solution.



[10, 30] scl: 5 by [−5, 5] scl: 1

You can apply the same methods used in solving square root equations to solving equations with roots of any index. Remember that to undo a square root, you square the expression. To undo an nth root, you must raise the expression to the nth power.

Study Tip

Alternative Method

To solve a radical equation, you can substitute a variable for the radical expression. In Example 3, let $A = 5n - 1$.

$$3A^{\frac{1}{3}} - 2 = 0$$
$$3A^{\frac{1}{3}} = 2$$
$$A^{\frac{1}{3}} = \frac{2}{3}$$
$$A = \frac{8}{27}$$
$$5n - 1 = \frac{8}{27}$$
$$n = \frac{7}{27}$$

Example 3 Cube Root Equation

Solve $3(5n - 1)^{\frac{1}{3}} - 2 = 0$.

In order to remove the $\frac{1}{3}$ power, or cube root, you must first isolate it and then raise each side of the equation to the third power.

$3(5n - 1)^{\frac{1}{3}} - 2 = 0$	Original equation
$3(5n - 1)^{\frac{1}{3}} = 2$	Add 2 to each side.
$(5n - 1)^{\frac{1}{3}} = \frac{2}{3}$	Divide each side by 3.
$\left[(5n - 1)^{\frac{1}{3}}\right]^3 = \left(\frac{2}{3}\right)^3$	Cube each side.
$5n - 1 = \frac{8}{27}$	Evaluate the cubes.
$5n = \frac{35}{27}$	Add 1 to each side.
$n = \frac{7}{27}$	Divide each side by 5.

CHECK

$3(5n - 1)^{\frac{1}{3}} - 2 = 0$	Original equation
$3\left(5 \cdot \frac{7}{27} - 1\right)^{\frac{1}{3}} - 2 \stackrel{?}{=} 0$	Replace n with $\frac{7}{27}$.
$3\left(\frac{8}{27}\right)^{\frac{1}{3}} - 2 \stackrel{?}{=} 0$	Simplify.
$3\left(\frac{2}{3}\right) - 2 \stackrel{?}{=} 0$	The cube root of $\frac{8}{27}$ is $\frac{2}{3}$.
$0 = 0$ ✓	Subtract.

The solution is $\frac{7}{27}$.

SOLVE RADICAL INEQUALITIES You can use what you know about radical equations to help solve radical inequalities. A **radical inequality** is an inequality that has a variable in a radicand.

Example 4 Radical Inequality

Solve $2 + \sqrt{4x - 4} \leq 6$.

Since the radicand of a square root must be greater than or equal to zero, first solve $4x - 4 \geq 0$ to identify the values of x for which the left side of the given inequality is defined.

$$4x - 4 \geq 0$$
$$4x \geq 4$$
$$x \geq 1$$

Now solve $2 + \sqrt{4x - 4} \leq 6$.

$2 + \sqrt{4x - 4} \leq 6$	Original inequality
$\sqrt{4x - 4} \leq 4$	Isolate the radical.
$4x - 4 \leq 16$	Eliminate the radical.
$4x \leq 20$	Add 4 to each side.
$x \leq 5$	Divide each side by 4.

It appears that $1 \leq x \leq 5$. You can test some x values to confirm the solution. Let $f(x) = 2 + \sqrt{4x - 4}$. Use three test values: one less than 1, one between 1 and 5, and one greater than 5. Organize the test values in a table.

$x = 0$	$x = 2$	$x = 7$
$f(0) = 2 + \sqrt{4(0) - 4}$ $= 2 + \sqrt{-4}$ Since $\sqrt{-4}$ is not a real number, the inequality is not satisfied.	$f(2) = 2 + \sqrt{4(2) - 4}$ $= 4$ Since $4 \leq 6$, the inequality is satisfied.	$f(7) = 2 + \sqrt{4(7) - 4}$ ≈ 6.90 Since $6.90 \not\leq 6$, the inequality is not satisfied.

The solution checks. Only values in the interval $1 \leq x \leq 5$ satisfy the inequality. You can summarize the solution with a number line.

Study Tip

Check Your Solution

You may also want to use a graphing calculator to check. Graph each side of the original inequality and examine the intersection.

Concept Summary — Solving Radical Inequalities

To solve radical inequalities, complete the following steps.

Step 1 If the index of the root is even, identify the values of the variable for which the radicand is nonnegative.

Step 2 Solve the inequality algebraically.

Step 3 Test values to check your solution.

Check for Understanding

Concept Check

1. **Explain** why you do not have to square each side to solve $2x + 1 = \sqrt{3}$. Then solve the equation.
2. **Show** how to solve $x - 6\sqrt{x} + 9 = 0$ by factoring. Name the properties of equality that you use.
3. **OPEN ENDED** Write an equation containing two radicals for which 1 is a solution.

Guided Practice

Solve each equation or inequality.

4. $\sqrt{4x+1}=3$

5. $4-(7-y)^{\frac{1}{2}}=0$

6. $1+\sqrt{x+2}=0$

7. $\sqrt{z-6}-3=0$

8. $\frac{1}{6}(12a)^{\frac{1}{3}}=1$

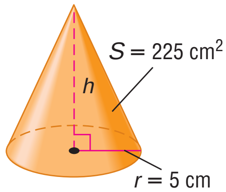
9. $\sqrt[3]{x-4}=3$

10. $\sqrt{2x+3}-4\leq 5$

11. $\sqrt{b+12}-\sqrt{b}>2$

Application

12. **GEOMETRY** The surface area S of a cone can be found by using $S=\pi r\sqrt{r^2+h^2}$, where r is the radius of the base and h is the height of the cone. Find the height of the cone.



Practice and Apply

Homework Help

For Exercises	See Examples
13–24, 29–32, 37–42	1–3
25–28, 33–36	4

Extra Practice
See page 839.

Solve each equation or inequality.

13. $\sqrt{x}=4$

14. $\sqrt{y}-7=0$

15. $a^{\frac{1}{2}}+9=0$

16. $2+4z^{\frac{1}{2}}=0$

17. $\sqrt[3]{c-1}=2$

18. $\sqrt[3]{5m+2}=3$

19. $7+\sqrt{4x+8}=9$

20. $5+\sqrt{4y-5}=12$

21. $(6n-5)^{\frac{1}{3}}+3=-2$

22. $(5x+7)^{\frac{1}{5}}+3=5$

23. $\sqrt{x-5}=\sqrt{2x-4}$

24. $\sqrt{2t-7}=\sqrt{t+2}$

25. $1+\sqrt{7x-3}>3$

26. $\sqrt{3x+6}+2\leq 5$

27. $-2+\sqrt{9-5x}\geq 6$

28. $6-\sqrt{2y+1}<3$

29. $\sqrt{x-6}-\sqrt{x}=3$

30. $\sqrt{y+21}-1=\sqrt{y+12}$

31. $\sqrt{b+1}=\sqrt{b+6}-1$

32. $\sqrt{4z+1}=3+\sqrt{4z-2}$

33. $\sqrt{2}-\sqrt{x+6}\leq-\sqrt{x}$

34. $\sqrt{a+9}-\sqrt{a}>\sqrt{3}$

35. $\sqrt{b-5}-\sqrt{b+7}\leq 4$

36. $\sqrt{c+5}+\sqrt{c+10}>2.5$

37. What is the solution of $2-\sqrt{x+6}=-1$?

38. Solve $\sqrt{2x+4}-4=2$.

39. **CONSTRUCTION** The minimum depth d in inches of a beam required to support a load of s pounds is given by the formula $d=\sqrt{\frac{s\ell}{576w}}$, where ℓ is the length of the beam in feet and w is the width in feet. Find the load that can be supported by a board that is 25 feet long, 2 feet wide, and 5 inches deep.

40. **AEROSPACE ENGINEERING** The radius r of the orbit of a satellite is given by $r=\sqrt[3]{\frac{GMt^2}{4\pi^2}}$, where G is the universal gravitational constant, M is the mass of the central object, and t is the time it takes the satellite to complete one orbit. Solve this formula for t.

41. **PHYSICS** When an object is dropped from the top of a 50-foot tall building, the object will be h feet above the ground after t seconds, where $\frac{\sqrt{50-h}}{4}=t$. How far above the ground will the object be after 1 second?

42. **HEALTH** **Use the information about health at the left.** A 70-kilogram person who is 1.8 meters tall has a ponderal index of about 2.29. How much weight could such a person gain and still have an index of at most 2.5?

43. **CRITICAL THINKING** Explain how you know that $\sqrt{x+2}+\sqrt{2x-3}=-1$ has no real solution without trying to solve it.

44. ***WRITING IN MATH*** Answer the question that was posed at the beginning of the lesson.

How do radical equations apply to manufacturing?

Include the following in your answer:
- the equation $C = 10n^{\frac{2}{3}} + 1500$ rewritten as a radical equation, and
- a step-by-step explanation of how to determine the maximum number of chips the company could make for $10,000.

More About. . .

Health

A ponderal index p is a measure of a person's body based on height h in meters and mass m in kilograms. One such formula is $p = \frac{\sqrt[3]{m}}{h}$.

Source: *A Dictionary of Food and Nutrition*

Standardized Test Practice

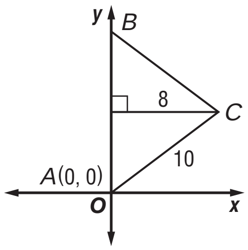
45. If $\sqrt{x+5}+1=4$, what is the value of x?

Ⓐ -4 Ⓑ 0 Ⓒ 2 Ⓓ 4

46. Side $\overline{AC}$ of triangle ABC contains which of the following points?

Ⓐ (3, 4) Ⓑ (3, 5) Ⓒ (4, 3)

Ⓓ (4, 5) Ⓔ (4, 6)



Maintain Your Skills

Mixed Review

Write each radical using rational exponents. *(Lesson 5-7)*

47. $\sqrt[7]{5^3}$ 48. $\sqrt{x+7}$ 49. $\left(\sqrt[3]{x^2+1}\right)^2$

Simplify. *(Lesson 5-6)*

50. $\sqrt{72x^6y^3}$ 51. $\frac{1}{\sqrt[3]{10}}$ 52. $\left(5-\sqrt{3}\right)^2$

53. **BUSINESS** A dry cleaner ordered 7 drums of two different types of cleaning fluid. One type cost $30 per drum, and the other type cost $20 per drum. The total cost was $160. How much of each type of fluid did the company order? Write a system of equations and solve by graphing. *(Lesson 3-1)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify each expression.**

*(To review **binomials**, see Lesson 5-2.)*

54. $(5+2x)+(-1-x)$ 55. $(-3-2y)+(4+y)$

56. $(4+x)-(2-3x)$ 57. $(-7-3x)-(4-3x)$

58. $(1+z)(4+2z)$ 59. $(-3-4x)(1+2x)$

Graphing Calculator

A Follow-Up of Lesson 5-8

Solving Radical Equations and Inequalities by Graphing

You can use a TI-83 Plus to solve radical equations and inequalities. One way to do this is by rewriting the equation or inequality so that one side is 0 and then using the zero feature on the calculator.

Solve $\sqrt{x} + \sqrt{x + 2} = 3$.

Step 1 *Rewrite the equation.*

- Subtract 3 from each side of the equation to obtain $\sqrt{x} + \sqrt{x + 2} - 3 = 0$.
- Enter the function $y = \sqrt{x} + \sqrt{x + 2} - 3$ in the Y= list.

KEYSTROKES: *Review entering a function on page 128.*

Step 2 *Use a table.*

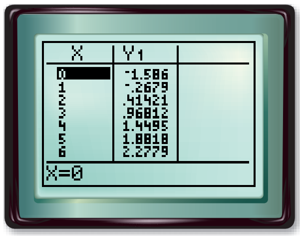
- You can use the **TABLE** function to locate intervals where the solution(s) lie. First, enter the starting value and the interval for the table.

KEYSTROKES: 2nd [TBLSET] 0 ENTER 1 ENTER

Step 3 *Estimate the solution.*

- Complete the table and estimate the solution(s).

KEYSTROKES: 2nd [TABLE]

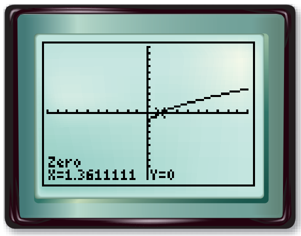


Since the function changes sign from negative to positive between $x = 1$ and $x = 2$, there is a solution between 1 and 2.

Step 4 *Use the zero feature.*

- Graph, then select zero from the **CALC** menu.

KEYSTROKES: GRAPH 2nd [CALC] 2



[−10, 10] scl: 1 by [−10, 10] scl: 1

Place the cursor to the left of the zero and press ENTER for the Left Bound. Then place the cursor to the right of the zero and press ENTER for the Right Bound. Press ENTER to solve.

The solution is about 1.36. This agrees with the estimate made by using the **TABLE**.

www.algebra2.com/other_calculator_keystrokes

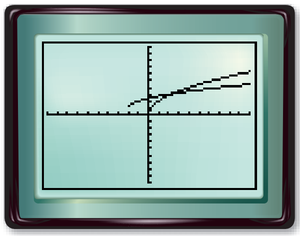
Investigation

Instead of rewriting an equation or inequality so that one side is 0, you can also treat each side of the equation or inequality as a separate function and graph both.

Solve $2\sqrt{x} > \sqrt{x+2} + 1$.

Step 1 *Graph each side of the inequality.*

- In the Y= list, enter $y_1 = 2\sqrt{x}$ and $y_2 = \sqrt{x+2} + 1$. Then press GRAPH.



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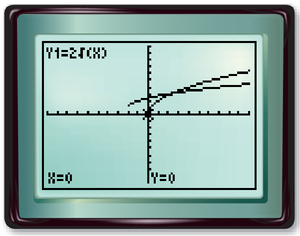
Step 2 *Use the trace feature.*

- Press . You can use ▲ or ▼ to switch the cursor between the two curves.



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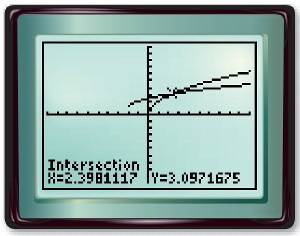
The calculator screen above shows that, for points to the left of where the curves cross, $Y_1 < Y_2$ or $2\sqrt{x} < \sqrt{x+2} + 1$. To solve the original inequality, you must find points for which $Y_1 > Y_2$. These are the points to the right of where the curves cross.

Step 3 *Use the intersect feature.*

- You can use the INTERSECT feature on the CALC menu to approximate the x-coordinate of the point at which the curves cross.

  KEYSTROKES: 2nd [CALC] 5

- Press ENTER for each of First curve?, Second curve?, and Guess?.



[−10, 10] scl: 1 by [−10, 10] scl: 1

The calculator screen shows that the x-coordinate of the point at which the curves cross is about 2.40. Therefore, the solution of the inequality is about $x > 2.40$. *Use the symbol $>$ instead of $\geq$ in the solution because the symbol in the original inequality is $>$.*

Exercises

Solve each equation or inequality.

1. $\sqrt{x+4} = 3$

2. $\sqrt{3x-5} = 1$

3. $\sqrt{x+5} = \sqrt{3x+4}$

4. $\sqrt{x+3} + \sqrt{x-2} = 4$

5. $\sqrt{3x-7} = \sqrt{2x-2} - 1$

6. $\sqrt{x+8} - 1 = \sqrt{x+2}$

7. $\sqrt{x-3} \geq 2$

8. $\sqrt{x+3} > 2\sqrt{x}$

9. $\sqrt{x} + \sqrt{x-1} < 4$

10. Explain how you could apply the technique in the first example to solving an inequality.

5-9 Complex Numbers

What You'll Learn

- Add and subtract complex numbers.
- Multiply and divide complex numbers.

How do complex numbers apply to polynomial equations?

Consider the equation $2x^2 + 2 = 0$. If you solve this equation for x^2, the result is $x^2 = -1$. Since there is no real number whose square is -1, the equation has no real solutions. French mathematician René Descartes (1596–1650) proposed that a number i be defined such that $i^2 = -1$.

Vocabulary

- imaginary unit
- pure imaginary number
- complex number
- absolute value
- complex conjugates

ADD AND SUBTRACT COMPLEX NUMBERS Since i is defined to have the property that $i^2 = -1$, the number i is the principal square root of -1; that is, $i = \sqrt{-1}$. i is called the **imaginary unit**. Numbers of the form $3i$, $-5i$, and $i\sqrt{2}$ are called **pure imaginary numbers**. Pure imaginary numbers are square roots of negative real numbers. For any positive real number b, $\sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1}$ or bi.

Example 1 Square Roots of Negative Numbers

Simplify.

a. $\sqrt{-18}$

$$\sqrt{-18} = \sqrt{-1 \cdot 3^2 \cdot 2}$$
$$= \sqrt{-1} \cdot \sqrt{3^2} \cdot \sqrt{2}$$
$$= i \cdot 3 \cdot \sqrt{2} \text{ or } 3i\sqrt{2}$$

b. $\sqrt{-125x^5}$

$$\sqrt{-125x^5} = \sqrt{-1 \cdot 5^2 \cdot x^4 \cdot 5x}$$
$$= \sqrt{-1} \cdot \sqrt{5^2} \cdot \sqrt{x^4} \cdot \sqrt{5x}$$
$$= i \cdot 5 \cdot x^2 \cdot \sqrt{5x} \text{ or } 5ix^2\sqrt{5x}$$

Study Tip

Reading Math

i is usually written before radical symbols to make it clear that it is not under the radical.

The Commutative and Associative Properties of Multiplication hold true for pure imaginary numbers.

Example 2 Multiply Pure Imaginary Numbers

Simplify.

a. $-2i \cdot 7i$

$$-2i \cdot 7i = -14i^2$$
$$= -14(-1) \quad i^2 = -1$$
$$= 14$$

b. $\sqrt{-10} \cdot \sqrt{-15}$

$$\sqrt{-10} \cdot \sqrt{-15} = i\sqrt{10} \cdot i\sqrt{15}$$
$$= i^2\sqrt{150}$$
$$= -1 \cdot \sqrt{25} \cdot \sqrt{6}$$
$$= -5\sqrt{6}$$

Example 3 Simplify a Power of i

Simplify i^{45}.

$i^{45} = i \cdot i^{44}$ Multiplying powers

$= i \cdot (i^2)^{22}$ Power of a Power

$= i \cdot (-1)^{22}$ $i^2 = -1$

$= i \cdot 1$ or i $(-1)^{22} = 1$

The solutions of some equations involve pure imaginary numbers.

Example 4 Equation with Imaginary Solutions

Solve $3x^2 + 48 = 0$.

$3x^2 + 48 = 0$	Original equation
$3x^2 = -48$	Subtract 48 from each side.
$x^2 = -16$	Divide each side by 3.
$x = \pm\sqrt{-16}$	Take the square root of each side.
$x = \pm 4i$	$\sqrt{-16} = \sqrt{16} \cdot \sqrt{-1}$

Study Tip

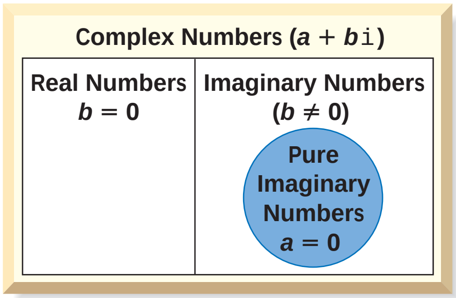
Quadratic Solutions
Quadratic equations always have complex solutions. If the discriminant is:
- negative, there are two imaginary roots,
- zero, there are two equal real roots, or
- positive, there are two unequal real roots.

Consider an expression such as $5 + 2i$. Since 5 is a real number and $2i$ is a pure imaginary number, the terms are not like terms and cannot be combined. This type of expression is called a **complex number**.

Key Concept — Complex Numbers

- **Words** A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit. a is called the real part, and b is called the imaginary part.
- **Examples** $7 + 4i$ and $2 - 6i = 2 + (-6)i$

The Venn diagram at the right shows the complex number system.
- If $b = 0$, the complex number is a real number.
- If $b \neq 0$, the complex number is imaginary.
- If $a = 0$, the complex number is a pure imaginary number.



Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. That is, $a + bi = c + di$ if and only if $a = c$ and $b = d$.

Example 5 Equate Complex Numbers

Find the values of x and y that make the equation $2x - 3 + (y - 4)i = 3 + 2i$ true.

Set the real parts equal to each other and the imaginary parts equal to each other.

$2x - 3 = 3$	Real parts
$2x = 6$	Add 3 to each side.
$x = 3$	Divide each side by 2.
$y - 4 = 2$	Imaginary parts
$y = 6$	Add 4 to each side.

Study Tip

Reading Math
The form $a + bi$ is sometimes called the **standard form** of a complex number.

www.algebra2.com/extra_examples

CONTENTS

To add or subtract complex numbers, combine like terms. That is, combine the real parts and combine the imaginary parts.

Example 6 Add and Subtract Complex Numbers

Simplify.

a. $(6 - 4i) + (1 + 3i)$

$(6 - 4i) + (1 + 3i) = (6 + 1) + (-4 + 3)i$ — Commutative and Associative Properties

$= 7 - i$ — Simplify.

b. $(3 - 2i) - (5 - 4i)$

$(3 - 2i) - (5 - 4i) = (3 - 5) + [-2 - (-4)]i$ — Commutative and Associative Properties

$= -2 + 2i$ — Simplify.

You can model the addition and subtraction of complex numbers geometrically.

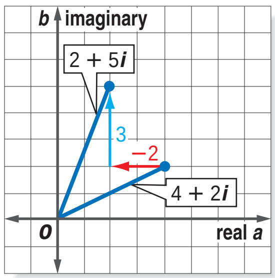
Algebra Activity

Adding Complex Numbers

You can model the addition of complex numbers on a coordinate plane. The horizontal axis represents the real part a of the complex number, and the vertical axis represents the imaginary part b of the complex number.

Use a coordinate plane to find $(4 + 2i) + (-2 + 3i)$.

- Create a coordinate plane and label the axes appropriately.
- Graph $4 + 2i$ by drawing a segment from the origin to (4, 2) on the coordinate plane.
- Represent the addition of $-2 + 3i$ by moving 2 units to the left and 3 units up from (4, 2).
- You end at the point (2, 5), which represents the complex number $2 + 5i$.
  So, $(4 + 2i) + (-2 + 3i) = 2 + 5i$.



Model and Analyze

1. Model $(-3 + 2i) + (4 - i)$ on a coordinate plane.

2. Describe how you could model the difference $(-3 + 2i) - (4 - i)$ on a coordinate plane.

3. The **absolute value** of a complex number is the distance from the origin to the point representing that complex number in a coordinate plane. Refer to the graph above. Find the absolute value of $2 + 5i$.

4. Find an expression for the absolute value of $a + bi$.

MULTIPLY AND DIVIDE COMPLEX NUMBERS Complex numbers are used with electricity. In a circuit with alternating current, the voltage, current, and impedance, or hindrance to current, can be represented by complex numbers.

You can use the FOIL method to multiply complex numbers.

Example 7 Multiply Complex Numbers

ELECTRICITY **In an AC circuit, the voltage E, current I, and impedance Z are related by the formula $E = I \cdot Z$. Find the voltage in a circuit with current $1 + 3j$ amps and impedance $7 - 5j$ ohms.**

$E = I \cdot Z$	Electricity formula
$= (1 + 3j) \cdot (7 - 5j)$	$I = 1 + 3j$, $Z = 7 - 5j$
$= 1(7) + 1(-5j) + (3j)7 + 3j(-5j)$	FOIL
$= 7 - 5j + 21j - 15j^2$	Multiply.
$= 7 + 16j - 15(-1)$	$j^2 = -1$
$= 22 + 16j$	Add.

The voltage is $22 + 16j$ volts.

Study Tip

Reading Math

Electrical engineers use j as the imaginary unit to avoid confusion with the I for current.

Two complex numbers of the form $a + bi$ and $a - bi$ are called **complex conjugates**. The product of complex conjugates is always a real number. For example, $(2 + 3i)(2 - 3i) = 4 - 6i + 6i + 9$ or 13. You can use this fact to simplify the quotient of two complex numbers.

Example 8 Divide Complex Numbers

Simplify.

a. $\frac{3i}{2 + 4i}$

$\frac{3i}{2 + 4i} = \frac{3i}{2 + 4i} \cdot \frac{2 - 4i}{2 - 4i}$	$2 + 4i$ and $2 - 4i$ are conjugates.
$= \frac{6i - 12i^2}{4 - 16i^2}$	Multiply.
$= \frac{6i + 12}{20}$	$i^2 = -1$
$= \frac{3}{5} + \frac{3}{10}i$	Standard form

b. $\frac{5 + i}{2i}$

$\frac{5 + i}{2i} = \frac{5 + i}{2i} \cdot \frac{i}{i}$	Why multiply by $\frac{i}{i}$ intead of $\frac{-2i}{-2i}$?
$= \frac{5i + i^2}{2i^2}$	Multiply.
$= \frac{5i - 1}{-2}$	$i^2 = -1$
$= \frac{1}{2} - \frac{5}{2}i$	Standard form

Check for Understanding

Concept Check

1. **Determine** if each statement is *true* or *false*. If false, find a counterexample.
   a. Every real number is a complex number.
   b. Every imaginary number is a complex number.
2. **Decide** which of the properties of a field and the properties of equality that the set of complex numbers satisfies.
3. **OPEN ENDED** Write two complex numbers whose product is 10.

Study Tip

Look Back

Refer to Chapter 1 to review the properties of fields and the properties of equality.

Guided Practice

Simplify.

4. $\sqrt{-36}$
5. $\sqrt{-50x^2y^2}$
6. $(6i)(-2i)$
7. $5\sqrt{-24} \cdot 3\sqrt{-18}$
8. i^{29}
9. $(8 + 6i) - (2 + 3i)$
10. $(3 - 5i)(4 + 6i)$
11. $\frac{3 + i}{1 + 4i}$

Solve each equation.

12. $2x^2 + 18 = 0$ **13.** $4x^2 + 32 = 0$ **14.** $-5x^2 - 25 = 0$

Find the values of m and n that make each equation true.

15. $2m + (3n + 1)i = 6 - 8i$ **16.** $(2n - 5) + (-m - 2)i = 3 - 7i$

Application

17. **ELECTRICITY** The current in one part of a series circuit is $4 - j$ amps. The current in another part of the circuit is $6 + 4j$ amps. Add these complex numbers to find the total current in the circuit.

Practice and Apply

Homework Help

For Exercises	See Examples
18–21	1
22–25	2
26–29	3
30–33, 46, 47	6
34–37, 42, 43	7
38–41, 44, 45	8
48–55	4
56–61	5

Extra Practice
See page 839.

Simplify.

18. $\sqrt{-144}$ **19.** $\sqrt{-81}$ **20.** $\sqrt{-64x^4}$

21. $\sqrt{-100a^4b^2}$ **22.** $\sqrt{-13} \cdot \sqrt{-26}$ **23.** $\sqrt{-6} \cdot \sqrt{-24}$

24. $(-2i)(-6i)(4i)$ **25.** $3i(-5i)^2$ **26.** i^{13}

27. i^{24} **28.** i^{38} **29.** i^{63}

30. $(5 - 2i) + (4 + 4i)$ **31.** $(3 - 5i) + (3 + 5i)$ **32.** $(3 - 4i) - (1 - 4i)$

33. $(7 - 4i) - (3 + i)$ **34.** $(3 + 4i)(3 - 4i)$ **35.** $(1 - 4i)(2 + i)$

36. $(6 - 2i)(1 + i)$ **37.** $(-3 - i)(2 - 2i)$ **38.** $\frac{4i}{3 + i}$

39. $\frac{4}{5 + 3i}$ **40.** $\frac{10 + i}{4 - i}$ **41.** $\frac{2 - i}{3 - 4i}$

42. $(-5 + 2i)(6 - i)(4 + 3i)$ **43.** $(2 + i)(1 + 2i)(3 - 4i)$

44. $\frac{5 - i\sqrt{3}}{5 + i\sqrt{3}}$ **45.** $\frac{1 - i\sqrt{2}}{1 + i\sqrt{2}}$

46. Find the sum of $ix^2 - (2 + 3i)x + 2$ and $4x^2 + (5 + 2i)x - 4i$.

47. Simplify $[(3 + i)x^2 - ix + 4 + i] - [(-2 + 3i)x^2 + (1 - 2i)x - 3]$.

Solve each equation.

48. $5x^2 + 5 = 0$ **49.** $4x^2 + 64 = 0$

50. $2x^2 + 12 = 0$ **51.** $6x^2 + 72 = 0$

52. $-3x^2 - 9 = 0$ **53.** $-2x^2 - 80 = 0$

54. $\frac{2}{3}x^2 + 30 = 0$ **55.** $\frac{4}{5}x^2 + 1 = 0$

Find the values of m and n that make each equation true.

56. $8 + 15i = 2m + 3ni$ **57.** $(m + 1) + 3ni = 5 - 9i$

58. $(2m + 5) + (1 - n)i = -2 + 4i$ **59.** $(4 + n) + (3m - 7)i = 8 - 2i$

60. $(m + 2n) + (2m - n)i = 5 + 5i$ **61.** $(2m - 3n)i + (m + 4n) = 13 + 7i$

62. **ELECTRICITY** The impedance in one part of a series circuit is $3 + 4j$ ohms, and the impedance in another part of the circuit is $2 - 6j$. Add these complex numbers to find the total impedance in the circuit.

Career Choices

Electrical Engineering

The chips and circuits in computers are designed by electrical engineers.

Online Research
To learn more about electrical engineering, visit: www.algebra2.com/careers

ELECTRICAL ENGINEERING **For Exercises 63 and 64, use the formula $E = I \cdot Z$.**

63. The current in a circuit is $2 + 5j$ amps, and the impedance is $4 - j$ ohms. What is the voltage?

64. The voltage in a circuit is $14 - 8j$ volts, and the impedance is $2 - 3j$ ohms. What is the current?

65. **CRITICAL THINKING** Show that the order relation "$<$" does not make sense for the set of complex numbers. (*Hint*: Consider the two cases $i > 0$ and $i < 0$. In each case, multiply each side by i.)

66. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do complex numbers apply to polynomial equations?

Include the following in your answer:

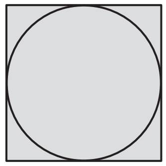
- how the a and c must be related if the equation $ax^2 + c = 0$ has complex solutions, and
- the solutions of the equation $2x^2 + 2 = 0$.

Standardized Test Practice

67. If $i^2 = -1$, then what is the value of i^{71}?

Ⓐ -1 Ⓑ 0 Ⓒ $-i$ Ⓓ i

68. The area of the square is 16 square units. What is the area of the circle?



Ⓐ 2π units^2 Ⓑ 12 units^2

Ⓒ 4π units^2 Ⓓ 16π units^2

Extending the Lesson

PATTERN OF POWERS OF i

69. Find the simplified forms of i^6, i^7, i^8, i^9, i^{10}, i^{11}, i^{12}, i^{13}, and i^{14}.

70. Explain how to use the exponent to determine the simplified form of any power of i.

Maintain Your Skills

Mixed Review

Solve each equation. *(Lesson 5-8)*

71. $\sqrt{2x + 1} = 5$

72. $\sqrt[3]{x - 3} + 1 = 3$

73. $\sqrt{x + 5} + \sqrt{x} = 5$

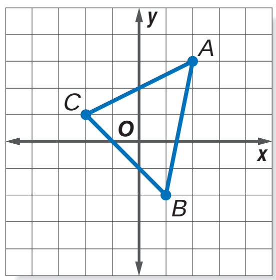
Simplify each expression. *(Lesson 5-7)*

74. $x^{-\frac{1}{5}} \cdot x^{\frac{2}{3}}$

75. $\left(y^{-\frac{1}{2}}\right)^{-\frac{2}{3}}$

76. $a^{-\frac{3}{4}}$

For Exercises 77–80, triangle ABC is reflected over the x-axis. *(Lesson 4-6)*



77. Write a vertex matrix for the triangle.

78. Write the reflection matrix.

79. Write the vertex matrix for $\triangle A'B'C'$.

80. Graph $\triangle A'B'C'$.

81. **FURNITURE** A new sofa, love seat, and coffee table cost \$2050. The sofa costs twice as much as the love seat. The sofa and the coffee table together cost \$1450. How much does each piece of furniture cost? *(Lesson 3-5)*

Graph each system of inequalities. *(Lesson 3-3)*

82. $y < x + 1$
$y > -2x - 2$

83. $x + y \geq 1$
$x - 2y \leq 4$

Find the slope of the line that passes through each pair of points. *(Lesson 2-3)*

84. $(-2, 1), (8, 2)$

85. $(4, -3), (5, -3)$

Chapter 5 Study Guide and Review

Vocabulary and Concept Check

absolute value (p. 272)
binomial (p. 229)
coefficient (p. 222)
complex conjugates (p. 273)
complex number (p. 271)
conjugates (p. 253)
constant (p. 222)
degree (p. 222)
dimensional analysis (p. 225)
extraneous solution (p. 263)
FOIL method (p. 230)
imaginary unit (p. 270)
like radical expressions (p. 252)
like terms (p. 229)
monomial (p. 222)
*n*th root (p. 245)
polynomial (p. 229)
power (p. 222)
principal root (p. 246)
pure imaginary number (p. 270)
radical equation (p. 263)
radical inequality (p. 264)
rationalizing the denominator (p. 251)
scientific notation (p. 225)
simplify (p. 222)
square root (p. 245)
standard notation (p. 225)
synthetic division (p. 234)
terms (p. 229)
trinomial (p. 229)

Choose a word or term from the list above that best completes each statement or phrase.

1. A number is expressed in _______ when it is in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer.
2. A shortcut method known as _______ is used to divide polynomials by binomials.
3. The _______ is used to multiply two binomials.
4. A(n) _______ is an expression that is a number, a variable, or the product of a number and one or more variables.
5. A solution of a transformed equation that is not a solution of the original equation is a(n) _______.
6. _______ are imaginary numbers of the form $a + bi$ and $a - bi$.
7. For any number a and b, if $a^2 = b$, then a is a(n) _______ of b.
8. A polynomial with three terms is known as a(n) _______.
9. When a number has more than one real root, the _______ is the nonnegative root.
10. i is called the _______.

Lesson-by-Lesson Review

5-1 Monomials

See pages 222–228.

Concept Summary

- The properties of powers for real numbers a and b and integers m and n are as follows.

$a^{-n} = \frac{1}{a^n}, a \neq 0$ $\qquad (a^m)^n = a^{mn}$

$a^m \cdot a^n = a^{m+n}$ $\qquad (ab)^m = a^m b^m$

$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ $\qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$

- Use scientific notation to represent very large or very small numbers.

Examples

1 **Simplify $(3x^4y^6)(-8x^3y)$.**

$(3x^4y^6)(-8x^3y) = (3)(-8)x^{4+3}y^{6+1}$ Commutative Property and products of powers

$= -24x^7y^7$ Simplify.

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2 **Express each number in scientific notation.**

a. 31,000

$31{,}000 = 3.1 \times 10{,}000$

$= 3.1 \times 10^4$ $\quad 10{,}000 = 10^4$

b. 0.007

$0.007 = 7 \times 0.001$

$= 7 \times 10^{-3}$ $\quad 0.001 = \frac{1}{1000}$ or $\frac{1}{10^3}$

Exercises **Simplify. Assume that no variable equals 0.**
See Examples 1–4 on pages 222–224.

11. $f^{-7} \cdot f^4$ **12.** $(3x^2)^3$ **13.** $(2y)(4xy^3)$ **14.** $\left(\frac{3}{5}c^2f\right)\left(\frac{4}{3}cd\right)^2$

Evaluate. Express the result in scientific notation. *See Examples 5–7 on page 225.*

15. $(2000)(85{,}000)$ **16.** $(0.0014)^2$ **17.** $\frac{5{,}400{,}000}{6000}$

5-2 Polynomials

See pages 229–232.

Concept Summary

- Add or subtract polynomials by combining like terms.
- Multiply polynomials by using the Distributive Property.
- Multiply binomials by using the FOIL method.

Examples

1 **Simplify $(5x^2 + 4x) - (3x^2 + 6x - 7)$.**

$5x^2 + 4x - (3x^2 + 6x - 7)$
$= 5x^2 + 4x - 3x^2 - 6x + 7$
$= (5x^2 - 3x^2) + (4x - 6x) + 7$
$= 2x^2 - 2x + 7$

2 **Find $(9k + 4)(7k - 6)$.**

$(9k + 4)(7k - 6)$
$= (9k)(7k) + (9k)(-6) + (4)(7k) + (4)(-6)$
$= 63k^2 - 54k + 28k - 24$
$= 63k^2 - 26k - 24$

Exercises **Simplify.** *See Examples 2–5 on pages 229 and 230.*

18. $(4c - 5) - (c + 11) + (-6c + 17)$ **19.** $(11x^2 + 13x - 15) - (7x^2 - 9x + 19)$
20. $-6m^2(3mn + 13m - 5n)$ **21.** $x^{-8}y^{10}(x^{11}y^{-9} + x^{10}y^{-6})$
22. $(d - 5)(d + 3)$ **23.** $(2a^2 + 6)^2$ **24.** $(2b - 3c)^3$

5-3 Dividing Polynomials

See pages 233–238.

Concept Summary

- Use the division algorithm or synthetic division to divide polynomials.

Example

Use synthetic division to find $(4x^4 - x^3 - 19x^2 + 11x - 2) \div (x - 2)$.

2	4	−1	−19	11	−2
		8	14	−10	2
	4	7	−5	1	0

→ The quotient is $4x^3 + 7x^2 - 5x + 1$.

Exercises **Simplify.** *See Examples 1–5 on pages 233–235.*

25. $(2x^4 - 6x^3 + x^2 - 3x - 3) \div (x - 3)$ **26.** $(10x^4 + 5x^3 + 4x^2 - 9) \div (x + 1)$
27. $(x^2 - 5x + 4) \div (x - 1)$ **28.** $(5x^4 + 18x^3 + 10x^2 + 3x) \div (x^2 + 3x)$

5-4 Factoring Polynomials

See pages 239–244.

Concept Summary

- You can factor polynomials using the GCF, grouping, or formulas involving squares and cubes.

Examples

1 **Factor $4x^3 - 6x^2 + 10x - 15$.**

$4x^3 - 6x^2 + 10x - 15 = (4x^3 - 6x^2) + (10x - 15)$ Group to find the GCF.

$= 2x^2(2x - 3) + 5(2x - 3)$ Factor the GCF of each binomial.

$= (2x^2 + 5)(2x - 3)$ Distributive Property

2 **Factor $3m^2 + m - 4$.**

Find two numbers whose product is $3(-4)$ or 12, and whose sum is 1. The two numbers must be 4 and -3 because $4(-3) = -12$ and $4 + (-3) = 1$.

$$3m^2 + m - 4 = 3m^2 + 4m - 3m - 4$$
$$= (3m^2 + 4m) - (3m + 4)$$
$$= m(3m + 4) + (-1)(3m + 4)$$
$$= (3m + 4)(m - 1)$$

Exercises **Factor completely. If the polynomial is not factorable, write *prime*.**

See Examples 1–3 on pages 239 and 241.

29. $200x^2 - 50$ **30.** $10a^3 - 20a^2 - 2a + 4$

31. $5w^3 - 20w^2 + 3w - 12$ **32.** $x^4 - 7x^3 + 12x^2$

33. $s^3 + 512$ **34.** $x^2 - 7x + 5$

5-5 Roots of Real Numbers

See pages 245–249.

Concept Summary

Real *n*th roots of *b*, $\sqrt[n]{b}$, or $-\sqrt[n]{b}$			
n	$\sqrt[n]{b}$ if $b > 0$	$\sqrt[n]{b}$ if $b < 0$	$\sqrt[n]{b}$ if $b = 0$
even	one positive root one negative root	no real roots	one real root, 0
odd	one positive root no negative roots	no positive roots one negative root	one real root, 0

Examples

1 **Simplify $\sqrt{81x^6}$.**

$\sqrt{81x^6} = \sqrt{(9x^3)^2}$ $81x^6 = (9x^3)^2$

$= 9|x^3|$ Use absolute value.

2 **Simplify $\sqrt[7]{2187x^{14}y^{35}}$.**

$\sqrt[7]{2187x^{14}y^{35}} = \sqrt[7]{(3x^2y^5)^7}$ $2187x^{14}y^{35} = (3x^2y^5)^7$

$= 3x^2y^5$ Evaluate.

Exercises **Simplify.** *See Examples 1 and 2 on pages 246 and 247.*

35. $\pm\sqrt{256}$ **36.** $\sqrt[3]{-216}$ **37.** $\sqrt{(-8)^2}$ **38.** $\sqrt[5]{c^5d^{15}}$

39. $\sqrt{(x^4 - 3)^2}$ **40.** $\sqrt[3]{(512 + x^2)^3}$ **41.** $\sqrt[4]{16m^8}$ **42.** $\sqrt{a^2 - 10a + 25}$

5-6 Radical Expressions

See pages 250–256.

Concept Summary

For any real numbers a and b and any integer $n > 1$,

- Product Property: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
- Quotient Property: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Example **Simplify $6\sqrt[5]{32m^3} \cdot 5\sqrt[5]{1024m^2}$.**

$6\sqrt[5]{32m^3} \cdot 5\sqrt[5]{1024m^2}$	$= 6 \cdot 5\sqrt[5]{(32m^3 \cdot 1024m^2)}$	Product Property of Radicals
	$= 30\sqrt[5]{2^5 \cdot 4^5 \cdot m^5}$	Factor into exponents of 5 if possible.
	$= 30\sqrt[5]{2^5} \cdot \sqrt[5]{4^5} \cdot \sqrt[5]{m^5}$	Product Property of Radicals
	$= 30 \cdot 2 \cdot 4 \cdot m$ or $240m$	Write the fifth roots.

Exercises **Simplify.** *See Examples 1–6 on pages 250–253.*

43. $\sqrt[6]{128}$
44. $\sqrt{5} + \sqrt{20}$
45. $5\sqrt{12} - 3\sqrt{75}$
46. $6\sqrt[5]{11} - 8\sqrt[5]{11}$
47. $(\sqrt{8} + \sqrt{12})^2$
48. $\sqrt{8} \cdot \sqrt{15} \cdot \sqrt{21}$
49. $\frac{\sqrt{243}}{\sqrt{3}}$
50. $\frac{1}{3 + \sqrt{5}}$
51. $\frac{\sqrt{10}}{4 + \sqrt{2}}$

5-7 Radical Exponents

See pages 257–262.

Concept Summary

- For any nonzero real number b, and any integers m and n, with $n > 1$,
  $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$

Examples

1 **Write $32^{\frac{4}{5}} \cdot 32^{\frac{2}{5}}$ in radical form.**

$32^{\frac{4}{5}} \cdot 32^{\frac{2}{5}}$	$= 32^{\frac{4}{5} + \frac{2}{5}}$	Product of powers
	$= 32^{\frac{6}{5}}$	Add.
	$= (2^5)^{\frac{6}{5}}$	$32 = 2^5$
	$= 2^6$ or 64	Power of a power

2 **Simplify $\frac{3x}{\sqrt[3]{z}}$.**

$\frac{3x}{\sqrt[3]{z}}$	$= \frac{3x}{z^{\frac{1}{3}}}$	Rational exponents
	$= \frac{3x}{z^{\frac{1}{3}}} \cdot \frac{z^{\frac{2}{3}}}{z^{\frac{2}{3}}}$	Rationalize the denominator.
	$= \frac{3xz^{\frac{2}{3}}}{z}$ or $\frac{3x\sqrt[3]{z^2}}{z}$	Rewrite in radical form.

Exercises **Evaluate.** *See Examples 3 and 5 on pages 258 and 259.*

52. $27^{-\frac{2}{3}}$
53. $9^{\frac{1}{3}} \cdot 9^{\frac{5}{3}}$
54. $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

Simplify. *See Example 5 on page 259.*

55. $\frac{1}{y^{\frac{2}{5}}}$
56. $\frac{xy}{\sqrt[3]{z}}$
57. $\frac{3x + 4x^2}{x^{-\frac{2}{3}}}$

Chapter 5 For More ...
• Extra Practice, see pages 836–839.
• Mixed Problem Solving, see page 866.

5-8 Radical Equations and Inequalities

See pages 263–267.

Concept Summary

- To solve a radical equation, isolate the radical. Then raise each side of the equation to a power equal to the index of the radical.

Example **Solve $\sqrt{3x - 8} + 1 = 3$.**

$\sqrt{3x - 8} + 1 = 3$	Original equation
$\sqrt{3x - 8} = 2$	Subtract 1 from each side.
$\left(\sqrt{3x - 8}\right)^2 = 2^2$	Square each side.
$3x - 8 = 4$	Evaluate the squares.
$x = 4$	Solve for *x*.

Exercises **Solve each equation.** *See Examples 1–3 on pages 263 and 264.*

58. $\sqrt{x} = 6$
59. $y^{\frac{1}{3}} - 7 = 0$
60. $(x - 2)^{\frac{3}{2}} = -8$
61. $\sqrt{x + 5} - 3 = 0$
62. $\sqrt{3t - 5} - 3 = 4$
63. $\sqrt{2x - 1} = 3$
64. $\sqrt[4]{2x - 1} = 2$
65. $\sqrt{y + 5} = \sqrt{2y - 3}$
66. $\sqrt{y + 1} + \sqrt{y - 4} = 5$

5-9 Complex Numbers

See pages 270–275.

Concept Summary

- $i^2 = -1$ and $i = \sqrt{-1}$
- Complex conjugates can be used to simplify quotients of complex numbers.

Examples

1 **Simplify $(15 - 2i) + (-11 + 5i)$.**

$(15 - 2i) + (-11 + 5i) = [15 + (-11)] + (-2 + 5)i$	Group the real and imaginary parts.
$= 4 + 3i$	Add.

2 **Simplify $\frac{7i}{2 + 3i}$.**

$\frac{7i}{2 + 3i} = \frac{7i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i}$	$2 + 3i$ and $2 - 3i$ are conjugates.
$= \frac{14i - 21i^2}{4 - 9i^2}$	Multiply.
$= \frac{21 + 14i}{13}$ or $\frac{21}{13} + \frac{14}{13}i$	$i^2 = -1$

Exercises **Simplify.** *See Examples 1–3 and 6–8 on pages 270, 272, and 273.*

67. $\sqrt{-64m^{12}}$
68. $(7 - 4i) - (-3 + 6i)$
69. $-6\sqrt{-9} \cdot 2\sqrt{-4}$
70. i^6
71. $(3 + 4i)(5 - 2i)$
72. $\left(\sqrt{6} + i\right)\left(\sqrt{6} - i\right)$
73. $\frac{1 + i}{1 - i}$
74. $\frac{4 - 3i}{1 + 2i}$
75. $\frac{3 - 9i}{4 + 2i}$

Chapter 5

Practice Test

Vocabulary and Concepts

Choose the term that best describes the shaded part of each trinomial.

1. $\boxed{2}x^2 - 3x + 4$
2. $4x^{\boxed{2}} - 6x - 3$
3. $9x^2 + 2x + \boxed{7}$

a. degree
b. constant term
c. coefficient

Skills and Applications

Simplify.

4. $(5b)^4(6c)^2$
5. $(13x - 1)(x + 3)$
6. $(2h - 6)^3$

Evaluate. Express the result in scientific notation.

7. $(3.16 \times 10^3)(24 \times 10^2)$
8. $\frac{7{,}200{,}000 \cdot 0.0011}{0.018}$

Simplify.

9. $(x^4 - x^3 - 10x^2 + 4x + 24) \div (x - 2)$
10. $(2x^3 + 9x^2 - 2x + 7) \div (x + 2)$

Factor completely. If the polynomial is not factorable, write *prime*.

11. $x^2 - 14x + 45$
12. $2r^2 + 3pr - 2p^2$
13. $x^2 + 2\sqrt{3}x + 3$

Simplify.

14. $\sqrt{175}$
15. $(5 + \sqrt{3})(7 - 2\sqrt{3})$
16. $3\sqrt{6} + 5\sqrt{54}$
17. $\frac{9}{5 - \sqrt{3}}$
18. $\left(9^{\frac{1}{2}} \cdot 9^{\frac{2}{3}}\right)^{\frac{1}{6}}$
19. $11^{\frac{1}{2}} \cdot 11^{\frac{7}{3}} \cdot 11^{\frac{1}{6}}$
20. $\sqrt[6]{256s^{11}t^{18}}$
21. $v^{-\frac{7}{11}}$
22. $\frac{b^{\frac{1}{2}}}{b^{\frac{3}{2}} - b^{\frac{1}{2}}}$

Solve each equation.

23. $\sqrt{b + 15} = \sqrt{3b + 1}$
24. $\sqrt{2x} = \sqrt{x - 4}$
25. $\sqrt[4]{y + 2} + 9 = 14$
26. $\sqrt[3]{2w - 1} + 11 = 18$
27. $\sqrt{4x + 28} = \sqrt{6x + 38}$
28. $1 + \sqrt{x + 5} = \sqrt{x + 12}$

Simplify.

29. $(5 - 2\boldsymbol{i}) - (8 - 11\boldsymbol{i})$
30. $(14 - 5\boldsymbol{i})^2$

31. **SKYDIVING** The approximate time t in seconds that it takes an object to fall a distance of d feet is given by $t = \sqrt{\frac{d}{16}}$. Suppose a parachutist falls 11 seconds before the parachute opens. How far does the parachutist fall during this time period?

32. **GEOMETRY** The area of a triangle with sides of length a, b, and c is given by $\sqrt{s(s - a)(s - b)(s - c)}$, where $s = \frac{1}{2}(a + b + c)$. If the lengths of the sides of a triangle are 6, 9, and 12 feet, what is the area of the triangle expressed in radical form?

33. **STANDARDIZED TEST PRACTICE** $2 + \left(x + \frac{1}{x}\right)^2 =$

Ⓐ 2
Ⓑ 4
Ⓒ $x^2 + \frac{1}{x^2}$
Ⓓ $x^2 + \frac{1}{x^2} + 4$

Chapter 5

Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. If $x^3 = 30$ and x is a real number, then x lies between which two consecutive integers?

(A) 2 and 3

(B) 3 and 4

(C) 4 and 5

(D) 5 and 6

2. If $12x + 7y = 19$ and $4x - y = 3$, then what is the value of $8x + 8y$?

(A) 2

(B) 8

(C) 16

(D) 22

3. For all positive integers n,

$\boxed{n} = n - 1$, if n is even and

$\boxed{n} = \frac{1}{2}(n + 1)$, if n is odd.

What is $\boxed{8} \times \boxed{13}$?

(A) 42

(B) 49

(C) 56

(D) 82

4. Let $x \circledast y = xy - y$ for all integers x and y. If $x \circledast y = 0$ and $y \neq 0$, what must x equal?

(A) -2

(B) -1

(C) 0

(D) 1

5. The sum of a number and its square is three times the number. What is the number?

(A) 0 only

(B) -2 only

(C) 2 only

(D) 0 or 2

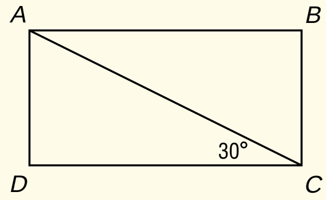
6. In rectangle $ABCD$, $\overline{AD}$ is 8 units long. What is the length of $\overline{AB}$ in units?

(A) 4

(B) 8

(C) $8\sqrt{3}$

(D) 16



7. The sum of two positive consecutive integers is s. In terms of s, what is the value of the greater integer?

(A) $\frac{s}{2} - 1$

(B) $\frac{s - 1}{2}$

(C) $\frac{s}{2}$

(D) $\frac{s + 1}{2}$

8. Latha, Renee, and Cindy scored a total of 30 goals for their soccer team this season. Latha scored three times as many goals as Renee. The combined number of goals scored by Latha and Cindy is four times the number scored by Renee. How many goals did Latha score?

(A) 5

(B) 6

(C) 18

(D) 20

9. If $s = t + 1$ and $t \geq 1$, then which of the following must be equal to $s^2 - t^2$?

(A) $(s - t)^2$

(B) $t^2 - 1$

(C) $s^2 - 1$

(D) $s + t$

Test-Taking Tip

Question 9
If you simplify an expression and do not find your answer among the given answer choices, follow these steps. First, check your answer. Then, compare your answer with each of the given answer choices to determine whether it is equivalent to any of the answer choices.

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 877–892.

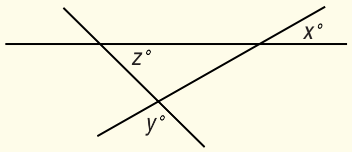
Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. Let a ✿ $b = a + \frac{1}{b}$, where $b \neq 0$. What is the value of 3 ✿ 4?

11. If $3x^2 = 27$, what is the value of $3x^4$?

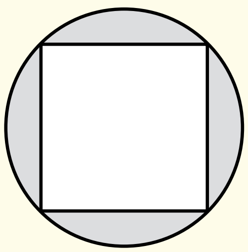
12. In the figure, if $x = 25$ and $z = 50$, what is the value of y?



13. For all positive integers n, let $\boxed{n}$ equal the greatest prime number that is a divisor of n. What does $\frac{\boxed{70}}{\boxed{27}}$ equal?

14. If $3x + 2y = 36$ and $\frac{5y}{3x} = 5$, then $x =$ __?__.

15. In the figure, a square with side of length $2\sqrt{2}$ is inscribed in a circle. If the area of the circle is $k\pi$, what is the exact value of k?



16. For all nonnegative numbers n, let $\boxed{n}$ be defined by $\boxed{n} = \frac{\sqrt{n}}{2}$. If $\boxed{n} = 4$, what is the value of n?

17. For the numbers a, b, and c, the average (arithmetic mean) is twice the median. If $a = 0$, and $a < b < c$, what is the value of $\frac{c}{b}$?

18. Write the numbers $\{\sqrt[3]{64}, \sqrt{18}, 4.1, 16^{\frac{1}{4}}\}$ in order from least to greatest. Show or describe your method.

19. Divide $x^3 - 7x^2 + 18x - 18$ by $x - 3$. Show each step of the process.

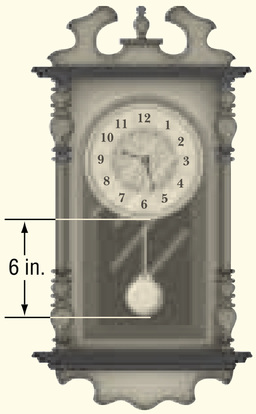
20. Simplify $\frac{x^2 - 2x - 8}{2x^2 - 9x + 4}$. Assume that the denominator is not equal to zero.

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

For Exercises 21–23, use the information below.

The period of a pendulum is the time it takes for the pendulum to make one complete swing back and forth. The formula $T = 2\pi\sqrt{\frac{L}{32}}$ gives the period T in seconds for a pendulum L feet long.



21. What is the period of the pendulum in the wall clock shown? Round to the nearest hundredth of a second.

22. Solve the formula for the length of the pendulum L in terms of the time T. Show each step of the process.

23. If you are building a grandfather clock and you want the pendulum to have a period of 2 seconds, how long should you make the pendulum? Round to the nearest tenth of a foot.