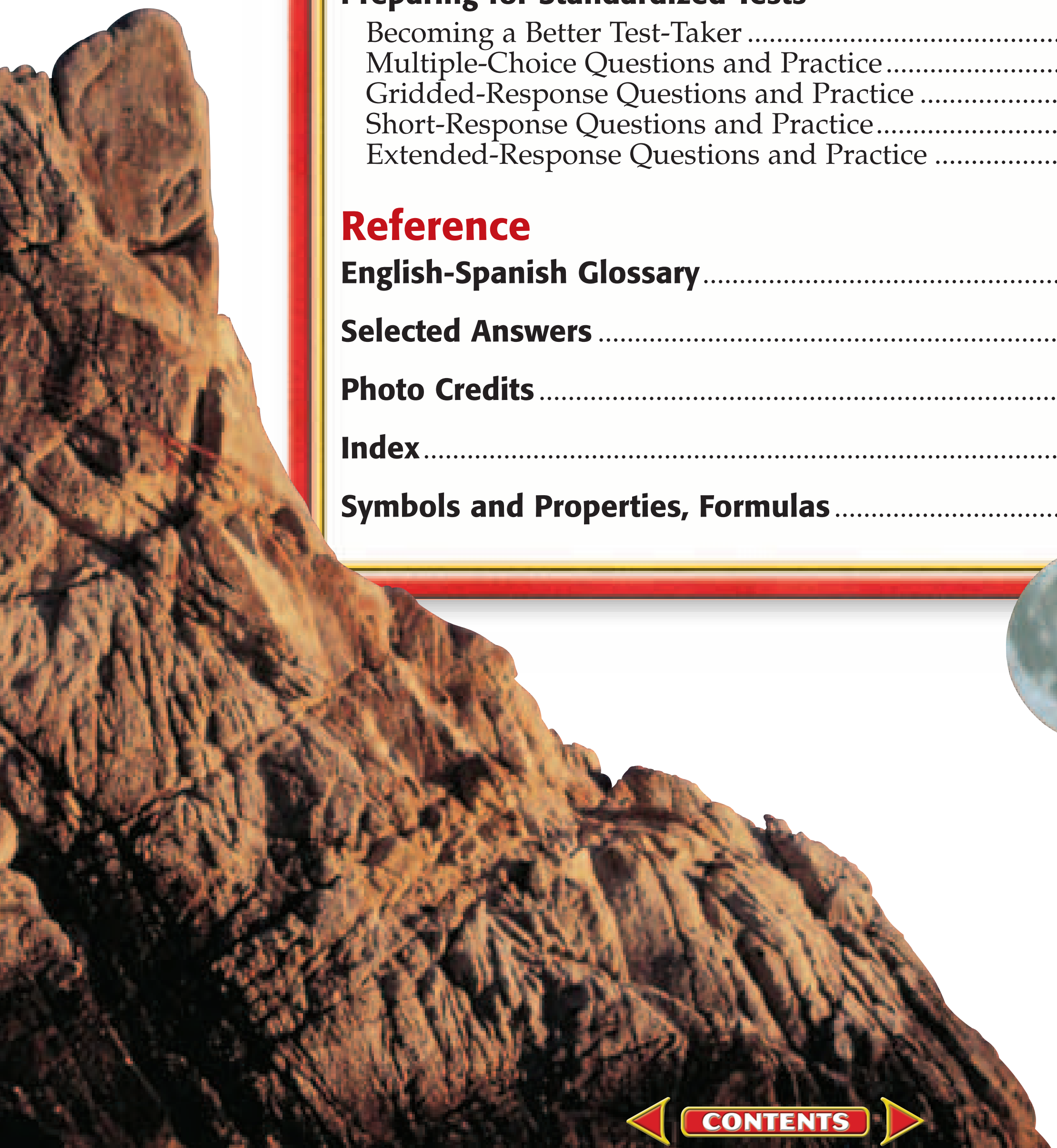
Student Handbook

Skills

Prerequisite Skills

Reference



Prerequisite Skills

1 Comparing and Ordering Real Numbers

- To determine which of two real numbers is greater, express each number as a decimal. Then compare the numbers.

Example 1 **Replace each ● with <, >, or = to make a true sentence.**

a. $\frac{4}{7}$ ● $0.\overline{5}$

$\frac{4}{7} \approx 0.57$ Round to the nearest hundredth.

$0.\overline{5} \approx 0.56$

Since $0.57 > 0.56$, $\frac{4}{7} > 0.\overline{5}$.

b. $\frac{1}{8}$ ● $\frac{1}{\sqrt{18}}$

$\frac{1}{8} = 0.125$ Use a calculator to find a rational approximation of .

$\frac{1}{\quad} \approx 0.236$

Since $0.125 < 0.236$, $\frac{1}{8} < \frac{1}{\quad}$.

- To order real numbers, first express each number as a decimal. Then write the decimals in order from least to greatest, and write the corresponding real numbers in the same order.

Example 2 **Order $2.\overline{54}$, $-\sqrt{6}$, $\frac{9}{4}$, $\frac{22}{-9}$ from least to greatest.**

$2.\overline{54} = 2.545454\ldots$ or about 2.55

$-\sqrt{6} = -2.44948974\ldots$ or about -2.45

$\frac{9}{4} = 2.25$

$\frac{22}{-9} = -2.4444\ldots$ or about -2.44

$-2.45 < -2.44 < 2.25 < 2.55$

Thus, the order from least to greatest is $-\sqrt{6}$, $\frac{22}{-9}$, $\frac{9}{4}$, $2.\overline{54}$.

Exercises **Replace each ● with <, >, or = to make a true sentence.**

1. $\frac{2}{3}$ ● 0.6

2. 0.35 ● $\frac{3}{8}$

3. $-\frac{3}{5}$ ● $-\frac{5}{7}$

4. $\frac{1}{39}$ ● $-\frac{2}{15}$

5. $0.\overline{4}$ ● $\frac{4}{9}$

6. $\frac{7}{11}$ ● $0.\overline{63}$

7. $\sqrt{5}$ ● $2\frac{1}{4}$

8. $\frac{1}{\sqrt{3}}$ ● $\frac{5}{9}$

9. $\frac{1}{\sqrt{2}}$ ● $\frac{11}{12}$

Order each set of numbers from least to greatest.

10. 0.1, 0.01, $\frac{3}{10}$, 0.2

11. $\frac{1}{3}$, 0.3, 0.4, $\frac{1}{4}$

12. 26.1, 26, 25.9, $\frac{181}{7}$

13. $\frac{8}{9}$, 0.89, $0.8\overline{9}$, $\frac{8}{11}$

14. 1.32, $-\sqrt{3}$, $\frac{4}{3}$, $\frac{-15}{11}$

15. $\frac{9}{2}$, $\frac{40}{-9}$, $-4.\overline{05}$, $\sqrt{18}$

16. $7.\overline{8}$, $8.\overline{7}$, $8.\overline{8}$, $8.\overline{78}$

17. 3.04, $4.0\overline{3}$, $3.0\overline{4}$, 4.03

Explain the difference between the following numbers and arrange them in order from least to greatest.

18. $3.\overline{54}$, $3.5\overline{4}$, 3.5, 3.54, $3.\overline{5}$

19. $2.\overline{987}$, $2.98\overline{7}$, 2.987, $2.9\overline{87}$, $2.\overline{98}$

20. 8.6, $8.6\overline{7}$, $8\frac{2}{3}$, 8.7676…

21. $-4.\overline{10}$, $-4\frac{1}{9}$, $-4.121231234\ldots$, $-4.1\overline{2}$

2 Factoring Polynomials

- Some polynomials can be factored using the Distributive Property.

Example 1 **Factor $4a^2 + 8a$.**

Find the GCF of $4a^2$ and $8a$.

$4a^2 = 2 \cdot 2 \cdot a \cdot a$

$8a = 2 \cdot 2 \cdot 2 \cdot a$

GCF: $2 \cdot 2 \cdot a$ or $4a$

$4a^2 + 8a = 4a(a) + 4a(2)$ Rewrite each term using the GCF.

$= 4a(a + 2)$ Distributive Property

Thus, the completely factored form of $4a^2 + 8a$ is $4a(a + 2)$.

- To factor quadratic trinomials of the form $x^2 + bx + c$, find two integers m and n whose product is equal to c and whose sum is equal to b. Then write $x^2 + bx + c$ using the pattern $(x + m)(x + n)$.

Example 2 **Factor each polynomial.**

a. $x^2 + 5x + 6$ ← Both b and c are positive.

In this trinomial, b is 5 and c is 6.

Find two numbers whose product is 6 and whose sum is 5.

Factors of 6	Sum of Factors
1, 6	7
2, 3	5

The correct factors are 2 and 3.

$x^2 + 5x + 6 = (x + m)(x + n)$ Write the pattern.

$= (x + 2)(x + 3)$ $m = 2$ and $n = 3$

CHECK Multiply the binomials to check the factorization.

$(x + 2)(x + 3) = x^2 + 3x + 2x + 2(3)$ FOIL

$= x^2 + 5x + 6$ ✓

b. $x^2 - 8x + 12$ ← b is negative and c is positive.

In this trinomial, $b = -8$ and $c = 12$. This means that $m + n$ is negative and mn is positive. So m and n must both be negative.

Factors of 12	Sum of Factors
−1, −12	−13
−2, −6	−8

The correct factors are −2 and −6.

$x^2 - 8x + 12 = (x + m)(x + n)$ Write the pattern.

$= [x + (-2)][x + (-6)]$ $m = -2$ and $n = -6$

$= (x - 2)(x - 6)$ Simplify.

c. $x^2 + 14x - 15$ ← b is positive and c is negative.

In this trinomial, $b = 14$ and $c = -15$. This means that $m + n$ is positive and mn is negative. So either m or n must be negative, but not both.

Factors of −15	Sum of Factors
1, −15	−14
−1, 15	14

The correct factors are −1 and 15.

$x^2 + 14x - 15 = (x + m)(x + n)$ Write the pattern.

$= [x + (-1)](x + 15)$ $m = -1$ and $n = 15$

$= (x - 1)(x + 15)$ Simplify.

- To factor quadratic trinomials of the form $ax^2 + bx + c$, find two integers m and n whose product is equal to ac and whose sum is equal to b. Write $ax^2 + bx + c$ using the pattern $ax^2 + mx + nx + c$. Then factor by grouping.

Example 3 **Factor $6x^2 + 7x - 3$.**

In this trinomial, $a = 6$, $b = 7$ and $c = -3$.
Find two numbers whose product is $6 \cdot (-3)$ or -18 and whose sum is 7.

Factors of −18	Sum of Factors
1, −18	−17
−1, 18	17
2, −9	−7
−2, 9	7

The correct factors are −2 and 9.

$6x^2 + 7x - 3 = 6x^2 + mx + nx - 3$	Write the pattern.
$= 6x^2 + (-2)x + 9x - 3$	$m = -2$ and $n = 9$
$= (6x^2 - 2x) + (9x - 3)$	Group terms with common factors.
$= 2x(3x - 1) + 3(3x - 1)$	Factor the GCF from each group.
$= (2x + 3)(3x - 1)$	Distributive Property

- Here are some special products.

Perfect Square Trinomials

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

$$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$$

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Example 4 **Factor each polynomial.**

a. $4x^2 + 20x + 25$

The first and last terms are perfect squares.
The middle term is equal to $2(2x)(5)$.
This is a perfect square trinomial of the form $(a + b)^2$.

$4x^2 + 20x + 25 = (2x)^2 + 2(2x)(5) + 5^2$	Write as $a^2 + 2ab + b^2$.
$= (2x + 5)^2$	Factor using the pattern.

b. $x^2 - 4$

This is a difference of squares.

$x^2 - 4 = x^2 - (2)^2$	Write in the form $a^2 - b^2$.
$= (x + 2)(x - 2)$	Factor the difference of squares.

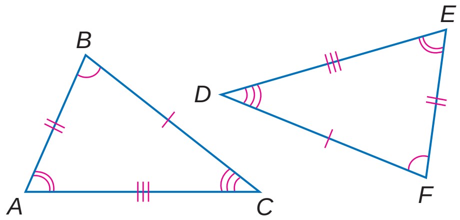
Exercises **Factor the following polynomials.**

1. $12x^2 + 4x$
2. $6x^2y + 2x$
3. $8ab^2 - 12ab$
4. $x^2 + 5x + 4$
5. $y^2 + 12y + 27$
6. $x^2 + 6x + 8$
7. $3y^2 + 13y + 4$
8. $7x^2 + 51x + 14$
9. $3x^2 + 28x + 32$
10. $x^2 - 5x + 6$
11. $y^2 - 5y + 4$
12. $6x^2 - 13x + 5$
13. $6a^2 - 50ab + 16b^2$
14. $11x^2 - 78x + 7$
15. $18x^2 - 31xy + 6y^2$
16. $x^2 + 4xy + 4y^2$
17. $9x^2 - 24x + 16$
18. $4a^2 + 12ab + 9b^2$
19. $x^2 - 144$
20. $4c^2 - 9$
21. $16y^2 - 1$
22. $25x^2 - 4y^2$
23. $36y^2 - 16$
24. $9a^2 - 49b^2$

3 Congruent and Similar Figures

Congruent figures have the same size and the same shape.

- Two polygons are congruent if their corresponding sides are congruent and their corresponding angles are congruent.



Congruent Angles	Congruent Sides
$\angle A \cong \angle E$	$\overline{AB} \cong \overline{EF}$
$\angle B \cong \angle F$	$\overline{BC} \cong \overline{FD}$
$\angle C \cong \angle D$	$\overline{AC} \cong \overline{ED}$

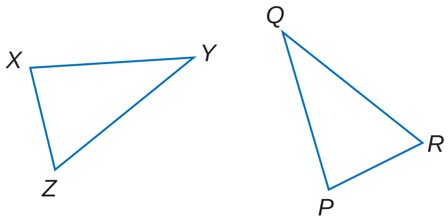
$\triangle ABC \cong \triangle EFD$

The order of the vertices indicates the corresponding parts.

Read the symbol $\cong$ as *is congruent to*.

Example 1 **If $\triangle XYZ \cong \triangle PQR$, name the congruent angles and sides.**

Name the pairs of congruent angles by looking at the order of the vertices in the statement $\triangle XYZ \cong \triangle PQR$.



So, $\angle X \cong \angle P$, $\angle Y \cong \angle Q$, and $\angle Z \cong \angle R$.

Since X corresponds to P, and Y corresponds to Q, $\overline{XY} \cong \overline{PQ}$.

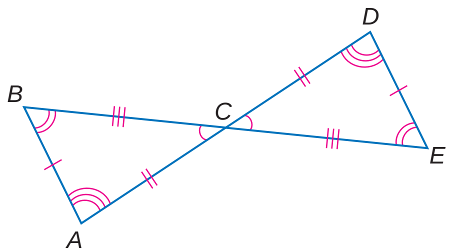
Since Y corresponds to Q, and Z corresponds to R, $\overline{YZ} \cong \overline{QR}$.

Since Z corresponds to R, and X corresponds to P, $\overline{ZX} \cong \overline{RP}$.

Example 2 **The corresponding parts of two congruent triangles are marked on the figure. Write a congruence statement for the two triangles.**

List the congruent angles and sides.

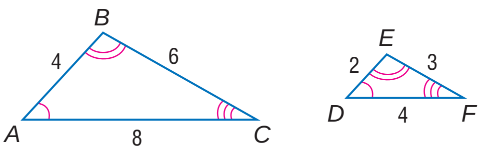
$\angle A \cong \angle D$	$\overline{AB} \cong \overline{DE}$
$\angle B \cong \angle E$	$\overline{AC} \cong \overline{DC}$
$\angle ACB \cong \angle DCE$	$\overline{BC} \cong \overline{EC}$



Match the vertices of the congruent angles. Therefore, $\triangle ABC \cong \triangle DEC$.

Similar figures have the same shape, but not necessarily the same size.

- In similar figures, corresponding angles are congruent, and the measures of corresponding sides are proportional. (They have equivalent ratios.)



Congruent Angles

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

Proportional Sides

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$\triangle ABC \sim \triangle DEF$

Read the symbol $\sim$ as *is similar to*.

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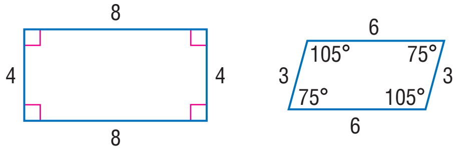
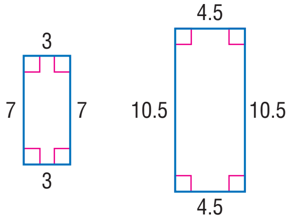
Example 3 **Determine whether the polygons are similar. Justify your answer.**

a.



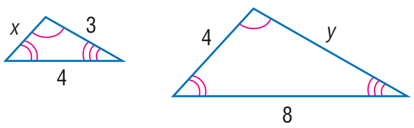
Since $\frac{4}{3} = \frac{8}{6} = \frac{4}{3} = \frac{8}{6}$, the measures of the sides of the polygons are proportional. However, the corresponding angles are not congruent. The polygons are not similar.

b.



Since $\frac{7}{10.5} = \frac{3}{4.5} = \frac{7}{10.5} = \frac{3}{4.5}$, the measures of the sides of the polygons are proportional. The corresponding angles are congruent. Therefore, the polygons are similar.

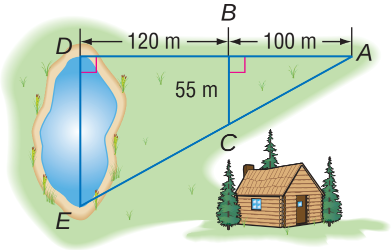
Example 4 **The triangles are similar. Find the values of x and y.**



Write proportions using corresponding parts. Then solve to find the missing measures.

$\frac{x}{4} = \frac{4}{8}$	Definition of similar polygons	$\frac{3}{y} = \frac{4}{8}$	Definition of similar polygons
$x(8) = 4(4)$	Cross products	$3(8) = y(4)$	Cross products
$8x = 16$	Simplify.	$24 = 4y$	Simplify.
$\frac{8x}{8} = \frac{16}{8}$	Divide each side by 8.	$\frac{24}{4} = \frac{4y}{4}$	Divide each side by 4.
$x = 2$	Simplify.	$6 = y$	Simplify.

Example 5 **CIVIL ENGINEERING** **The city of Mansfield plans to build a bridge across Pine Lake. Use the information in the diagram to find the distance across Pine Lake.**



$\triangle ABC \sim \triangle ADE$

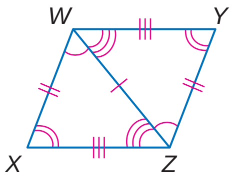
$\frac{AB}{AD} = \frac{BC}{DE}$	Definition of similar polygons
$\frac{100}{220} = \frac{55}{DE}$	$AB = 100$, $AD = 100 + 120 = 220$, $BC = 55$
$100DE = 220(55)$	Cross products
$100DE = 12{,}100$	Simplify.
$DE = 121$	Divide each side by 100.

The distance across the lake is 121 meters.

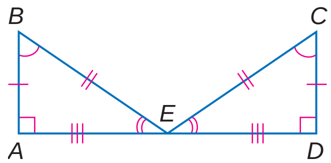
Exercises **If $\triangle GHI \cong \triangle JKL$, name the part that is congruent to each angle or segment.**

1. $\angle K$
2. $\overline{HI}$
3. $\angle I$
4. $\angle J$
5. $\overline{JK}$
6. $\overline{GI}$

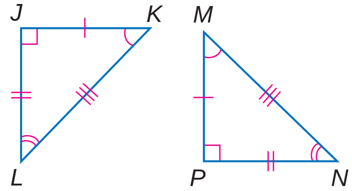
Complete each congruence statement.

7. 

$\triangle XWZ \cong \triangle$ __?__

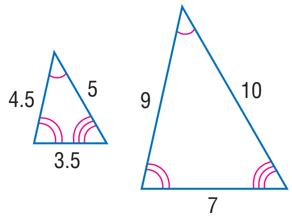
8. 

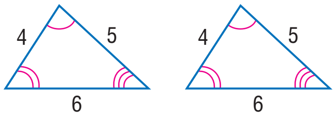
$\triangle ABE \cong \triangle$ __?__

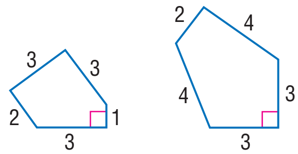
9. 

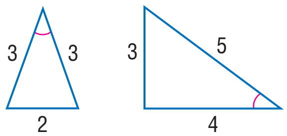
$\triangle JKL \cong \triangle$ __?__

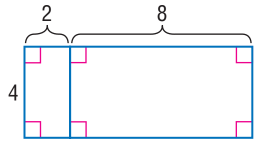
Determine whether each pair of figures is *similar*, *congruent*, or *neither*.

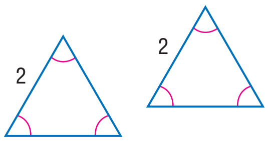
10. 

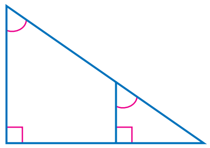
11. 

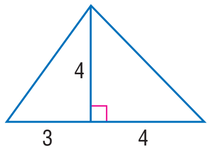
12. 

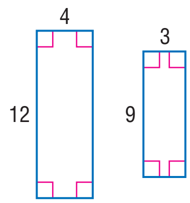
13. 

14. 

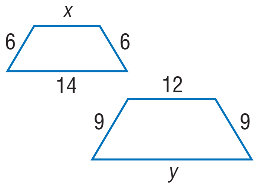
15. 

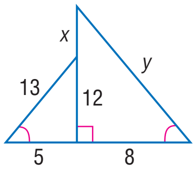
16. 

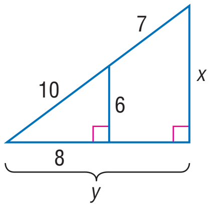
17. 

18. 

Each pair of polygons is similar. Find the values of *x* and *y*.

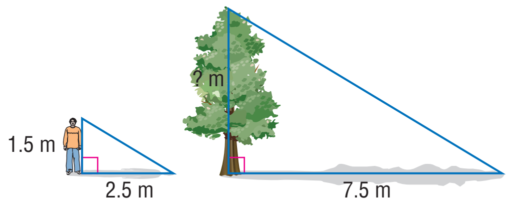
19. 

20. 

21. 

22. What are the dimensions of a scale drawing of a room that measures 10 feet by 12 feet if $\frac{1}{2}$ inch represents 1 foot?

23. **SHADOWS** On a sunny day, Jason measures the length of his shadow and the length of a tree's shadow. Use the figures at the right to find the height of the tree.



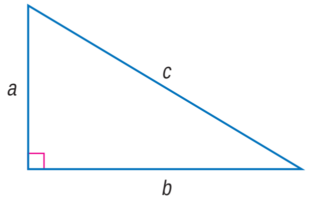
24. **PHOTOGRAPHY** A photo that is 4 inches wide by 6 inches long must be reduced to fit in a space 3 inches wide. How long will the reduced photo be?

25. **SURVEYING** Surveyors use instruments to measure objects that are too large or too far away to measure by hand. They can use the shadows that objects cast to find the height of the objects without measuring them. A surveyor finds that a telephone pole that is 25 feet tall is casting a shadow 20 feet long. A nearby building is casting a shadow 52 feet long. What is the height of the building?

4 Pythagorean Theorem

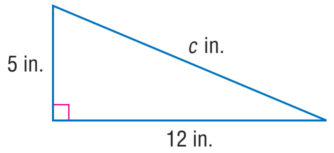
The **Pythagorean Theorem** states that in a right triangle, the square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the legs a and b.

That is, in any right triangle, $c^2 = a^2 + b^2$.



Example 1 **Find the length of the hypotenuse of each right triangle.**

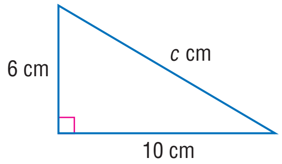
a.



$c^2 = a^2 + b^2$ Pythagorean Theorem
$c^2 = 5^2 + 12^2$ Replace a with 5 and b with 12.
$c^2 = 25 + 144$ Simplify.
$c^2 = 169$ Add.
$c = \sqrt{169}$ Take the square root of each side.
$c = 13$ Simplify.

The length of the hypotenuse is 13 inches.

b.

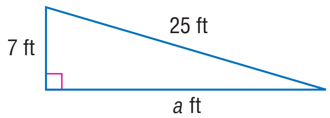


$c^2 = a^2 + b^2$ Pythagorean Theorem
$c^2 = 6^2 + 10^2$ Replace a with 6 and b with 10.
$c^2 = 36 + 100$ Simplify.
$c^2 = 136$ Add.
$c = \sqrt{136}$ Take the square root of each side.
$c \approx 11.7$ Use a calculator to find the square root of 136. Round to the nearest tenth.

To the nearest tenth, the length of the hypotenuse is 11.7 centimeters.

Example 2 **Find the length of the missing leg in each right triangle.**

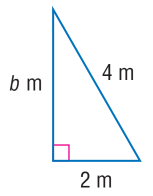
a.



$c^2 = a^2 + b^2$ Pythagorean Theorem
$25^2 = a^2 + 7^2$ Replace c with 25 and b with 7.
$625 = a^2 + 49$ Simplify.
$625 - 49 = a^2 + 49 - 49$ Subtract 49 from each side.
$576 = a^2$ Simplify.
$\sqrt{576} = a$ Take the square root of each side.
$24 = a$ Simplify.

The length of the leg is 24 feet.

b.



$c^2 = a^2 + b^2$	Pythagorean Theorem
$4^2 = 2^2 + b^2$	Replace *c* with 4 and *a* with 2.
$16 = 4 + b^2$	Simplify.
$12 = b^2$	Subtract 4 from each side.
$\sqrt{12} = b$	Take the square root of each side.
$3.5 \approx b$	Use a calculator to find the square root of 12. Round to the nearest tenth.

To the nearest tenth, the length of the leg is 3.5 meters.

Example 3 **The lengths of the three sides of a triangle are 5, 7, and 9 inches. Determine whether this triangle is a right triangle.**

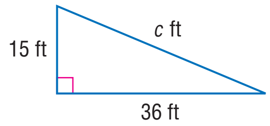
Since the longest side is 9 inches, use 9 as *c*, the measure of the hypotenuse.

$c^2 = a^2 + b^2$	Pythagorean Theorem
$9^2 \stackrel{?}{=} 5^2 + 7^2$	Replace *c* with 9, *a* with 5, and *b* with 7.
$81 \stackrel{?}{=} 25 + 49$	Evaluate 9^2, 5^2, and 7^2.
$81 \neq 74$	Simplify.

Since $c^2 \neq a^2 + b^2$, the triangle is *not* a right triangle.

Exercises **Find each missing measure. Round to the nearest tenth, if necessary.**

1.



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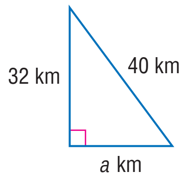

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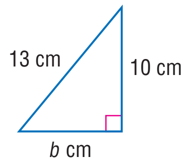

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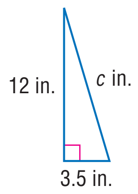

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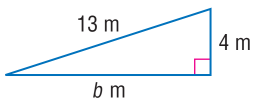

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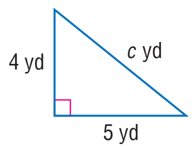


7. $a = 3, b = 4, c = ?$

8. $a = ?, b = 12, c = 13$

9. $a = 14, b = ?, c = 50$

10. $a = 2, b = 9, c = ?$

11. $a = 6, b = ?, c = 13$

12. $a = ?, b = 7, c = 11$

The lengths of three sides of a triangle are given. Determine whether each triangle is a right triangle.

13. 5 in., 7 in., 8 in.

14. 9 m, 12 m, 15 m

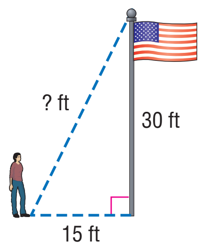
15. 6 cm, 7 cm, 12 cm

16. 11 ft, 12 ft, 16 ft

17. 10 yd, 24 yd, 26 yd

18. 11 km, 60 km, 61 km

19. FLAGPOLES Mai-Lin wants to find the distance from her feet to the top of the flagpole. If the flagpole is 30 feet tall and Mai-Lin is standing a distance of 15 feet from the flagpole, what is the distance from her feet to the top of the flagpole?



20. CONSTRUCTION The walls of the Downtown Recreation Center are being covered with paneling. The doorway into one room is 0.9 meters wide and 2.5 meters high. What is the width of the widest rectangular panel that can be taken through this doorway?

5 Mean, Median, and Mode

Mean, median, and mode are measures of central tendency that are often used to represent a set of data.

- To find the **mean**, find the sum of the data and divide by the number of items in the data set. (The mean is often called the average.)
- To find the **median**, arrange the data in numerical order. The median is the middle number. If there is an even number of data, the median is the mean of the two middle numbers.
- The **mode** is the number (or numbers) that appears most often in a set of data. If no item appears most often, the set has no mode.

Example 1 **Michelle is saving to buy a car. She saved $200 in June, $300 in July, $400 in August, and $150 in September. What was her mean (or average) monthly savings?**

$$\text{mean} = \frac{\text{sum of monthly savings}}{\text{number of months}}$$

$$= \frac{\$200 + \$300 + \$400 + \$150}{4}$$

$$= \frac{\$1050}{4} \text{ or } \$262.50$$

Michelle's mean monthly savings was $262.50.

Example 2 **Find the median of the data.**

Peter's Best Running Times	
Week	**Minutes to Run a Mile**
1	4.5
2	3.7
3	4.1
4	4.1
5	3.6
6	3.4

To find the median, order the numbers from least to greatest.
The median is in the middle.
3.4, 3.6, 3.7, 4.1, 4.1, 4.5

$$\frac{3.7 + 4.1}{2} = 3.9$$

There is an even number of data. Find the mean of the middle two.

Example 3 GOLF **Four players tied for first in the 2001 PGA Tour Championship. The scores for each player for each round are shown in the table below. What is the mode score?**

Player	Round 1	Round 2	Round 3	Round 4
Mike Weir	68	66	68	68
David Toms	73	66	64	67
Sergio Garcia	69	67	66	68
Ernie Els	69	68	65	68

Source: ESPN

The mode is the score that occurred most often. Since the score of 68 occurred 6 times, it is the mode of these data.

- The **range** of a set of data is the difference between the greatest and the least values of the set. It describes how a set of data varies.

Example 4 **Find the range of the data.**
{6, 11, 18, 4, 9, 15, 6, 3}

The greatest value is 18 and the least value is 3.
So, the range is 18 − 3 or 15.

Exercises **Find the mean, median, mode, and range for each set of data. Round to the nearest tenth if necessary.**

1. {2, 8, 12, 13, 15}

2. {66, 78, 78, 64, 34, 88}

3. {87, 95, 84, 89, 100, 82}

4. {99, 100, 85, 96, 94, 99}

5. {9.9, 9.9, 10, 9.9, 8.8, 9.5, 9.5}

6. {501, 503, 502, 502, 502, 504, 503, 503}

7. {7, 19, 15, 13, 11, 17, 9}

8. {6, 12, 21, 43, 1, 3, 13, 8}

9. {0.8, 0.04, 0.9, 1.1, 0.25}

10. $\left\{2\frac{1}{2}, 1\frac{7}{8}, 2\frac{5}{8}, 2\frac{3}{4}, 2\frac{1}{8}\right\}$

11. CHARITY The table shows the amounts collected by classes at Jackson High School. Find the mean, median, mode, and range of the data.

Amounts Collected for Charity	
Class	**Amount**
A	$150
B	$300
C	$55
D	$40
E	$10
F	$25
G	$200
H	$100

12. SCHOOL The table shows Pilar's grades in chemistry class for the semester. Find her mean, median, and mode scores, and the range of her scores.

Chemistry Grades	
Assignment	**Grade (out of 100)**
Homework	100
Electron Project	98
Test I	87
Atomic Mass Project	95
Test II	88
Phase Change Project	90
Test III	95

13. WEATHER The table shows the precipitation for the month of July in Cape Hatteras, North Carolina in various years. Find the mean, median, mode, and range of the data.

July Precipitation in Cape Hatteras, North Carolina												
Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Inches	4.23	8.58	5.28	2.03	3.93	1.08	9.54	4.94	10.85	2.66	6.04	3.26

Source: National Climatic Data Center

14. SCHOOL Kaitlyn's scores on her first five algebra tests are 88, 90, 91, 89, and 92. What test score must Kaitlyn earn on the sixth test so that her mean score will be at least 90?

15. GOLF Colin's average for three rounds of golf is 94. What is the highest score he can receive for the fourth round to have an average (mean) of 92?

16. SCHOOL Mika has a mean score of 21 on his first four Spanish quizzes. If each quiz is worth 25 points, what is the highest possible mean score he can have after the fifth quiz?

17. SCHOOL To earn a grade of B in math, Latisha must have an average (mean) score of at least 84 on five math tests. Her scores on the first three tests are 85, 89, and 82. What is the lowest total score that Latisha must have on the last two tests to earn a B test average?

6 Bar and Line Graphs

A **bar graph** compares different categories of data by showing each as a bar whose length is related to the frequency. A **double bar graph** compares two sets of data. Another way to represent data is by using a **line graph**. A line graph usually shows how data changes over a period of time.

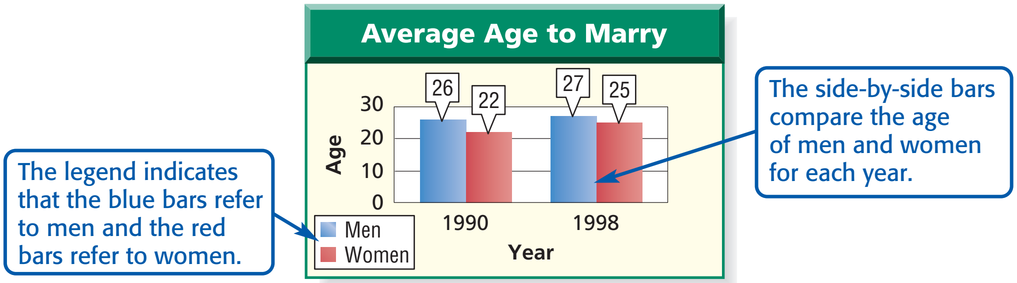
Example 1 MARRIAGE **The table shows the average age at which Americans marry for the first time. Make a double bar graph to display the data.**

Average Age to Marry		
Year	1990	1998
Men	26	27
Women	22	25

Source: U.S. Census Bureau

Step 1 Draw a horizontal and a vertical axis and label them as shown.

Step 2 Draw side-by-side bars to represent each category.



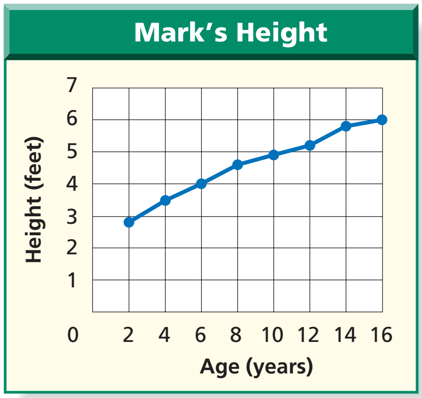
Example 2 HEALTH **The table shows Mark's height at 2-year intervals. Make a line graph to display the data.**

Age	2	4	6	8	10	12	14	16
Height (feet)	2.8	3.5	4.0	4.6	4.9	5.2	5.8	6

Step 1 Draw a horizontal and a vertical axis. Label them as shown.

Step 2 Plot the points.

Step 3 Draw a line connecting each pair of consecutive points.



Exercises

1. **HEALTH** The table below shows the life expectancy for Americans born in each year listed. Make a double-bar graph to display the data.

Life Expectancy		
Year of Birth	Male	Female
1980	70.0	77.5
1985	71.2	78.2
1990	71.8	78.8
1995	72.5	78.9
1998	73.9	79.4

2. **MONEY** The amount of money in Becky's savings account from August through March is shown in the table below. Make a line graph to display the data.

Month	Amount	Month	Amount
August	$300	December	$780
September	$400	January	$800
October	$700	February	$950
November	$780	March	$900

7 Stem-and-Leaf Plots

In a **stem-and-leaf plot**, data are organized in two columns. The greatest place value of the data is used for the stems. The next greatest place value forms the leaves. Stem-and-leaf plots are useful for organizing long lists of numbers.

Example

SCHOOL **Isabella has collected data on the GPAs (grade point average) of the 16 students in the art club. Display the data in a stem-and-leaf plot.**
{4.0, 3.9, 3.1, 3.9, 3.8, 3.7, 1.8, 2.6, 4.0, 3.9, 3.5, 3.3, 2.9, 2.5, 1.1, 3.5}

Step 1 Find the least and the greatest number. Then identify the greatest place-value digit in each number. In this case, ones.

least data: 1.1 — The least number has 1 in the ones place.

greatest data: 4.0 — The greatest number has 4 in the ones place.

Step 2 Draw a vertical line and write the stems from 1 to 4 to the left of the line.

Step 3 Write the leaves to the right of the line, with the corresponding stem. For example, write 0 to the right of 4 for 4.0.

Stem	Leaf
1	8 1
2	5 6 9
3	9 1 9 8 7 9 5 3 5
4	0 0

Step 4 Rearrange the leaves so they are ordered from least to greatest.

Step 5 Include a key or an explanation.

Stem	Leaf
1	1 8
2	5 6 9
3	1 3 5 5 7 8 9 9 9
4	0 0 *3|1 = 3.1*

Exercises

GAMES **For Exercises 1–4, use the following information.**
The stem-and-leaf plot at the right shows Charmaine's scores for her favorite computer game.

Stem	Leaf
9	0 0 0 1 3 4 5 5 7 8 8 8 9 9
10	0 3 4 4 5 6 9
11	0 3 9 9
12	1 2 6
13	0 *12|6 = 126*

1. What are Charmaine's highest and lowest scores?
2. Which score(s) occurred most frequently?
3. How many scores were above 115?
4. Has Charmaine ever scored 123?
5. **SCHOOL** The class scores on a 50-item test are shown in the table at the right. Make a stem-and-leaf plot of the data.

Test Scores					
45	15	30	40	28	35
39	29	38	18	43	49
46	44	48	35	36	30

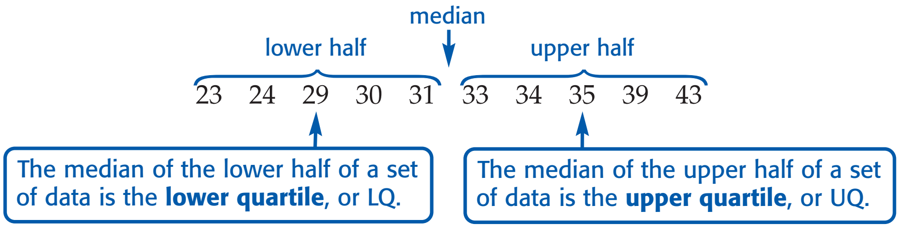
6. **GEOGRAPHY** The table shows the land area of each county in Wyoming. Round each area to the nearest hundred square miles and organize the data in a stem-and-leaf plot.

County	Area (mi^2)	County	Area (mi^2)	County	Area (mi^2)
Albany	4273	Hot Springs	2004	Sheridan	2523
Big Horn	3137	Johnson	4166	Sublette	4883
Campbell	4797	Laramie	2686	Sweetwater	10,425
Carbon	7896	Lincoln	4069	Teton	4008
Converse	4255	Natrona	5340	Unita	2082
Crook	2859	Niobrara	2626	Washakie	2240
Fremont	9182	Park	6942	Weston	2398
Goshen	2225	Platte	2085		

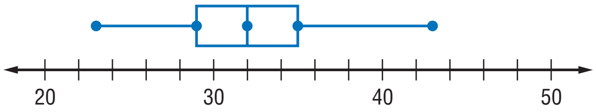
Source: *The World Almanac*

8 Box-and-Whisker Plots

In a set of data, **quartiles** are values that divide the data into four equal parts.



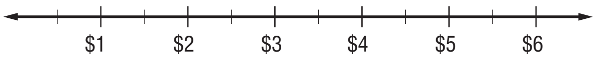
- To make a **box-and-whisker plot**, draw a box around the quartile values, and lines or *whiskers* to represent the values in the lower fourth of the data and the upper fourth of the data.



Example 1 **MONEY** **The amount spent in the cafeteria by 20 students is shown. Display the data in a box-and-whisker plot.**

Amount Spent			
\$2.00	\$2.00	\$1.00	\$4.00
\$1.00	\$2.50	\$2.50	\$2.00
\$2.50	\$1.00	\$4.00	\$2.50
\$3.50	\$2.00	\$3.00	\$2.50
\$4.00	\$4.00	\$5.50	\$1.50

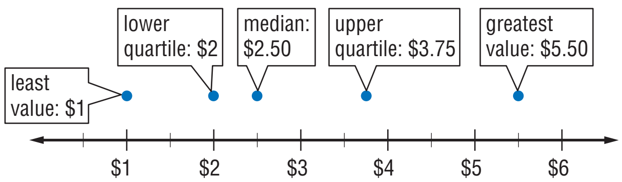
Step 1 Find the least and greatest number. Then draw a number line that covers the range of the data. In this case, the least value is 1 and the greatest value is 5.5.



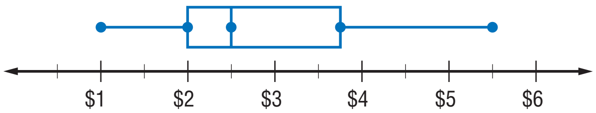
Step 2 Find the median, the extreme values, and the upper and lower quartiles. Mark these points above the number line.

1, 1, 1, 1.5, 2, 2, 2, 2, 2.5, 2.5, 2.5, 2.5, 2.5, 3, 3.5, 4, 4, 4, 4, 5.5

$LQ = \frac{2 + 2}{2}$ or 2 $M = \frac{2.5 + 2.5}{2}$ or 2.5 $UQ = \frac{3.5 + 4}{2}$ or 3.75



Step 3 Draw a box and the whiskers.



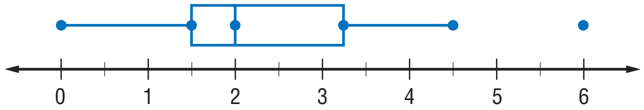
- The **interquartile range (IQR)** is the range of the middle half of the data and contains 50% of the data in the set.

$$\text{Interquartile range} = UQ - LQ$$

The interquartile range of the data in Example 1 is 3.75 − 2 or 1.75.

- An **outlier** is any element of a set that is at least 1.5 interquartile ranges less than the lower quartile or greater than the upper quartile. The whisker representing the data is drawn from the box to the least or greatest value that is not an outlier.

Example 2 **SCHOOL** **The number of hours José studied each day for the last month is shown in the box-and-whiskers plot below.**



a. What percent of the data lies between 1.5 and 3.25?
The value 1.5 is the lower quartile and 3.25 is the upper quartile. The values between the lower and upper quartiles represent 50% of the data.

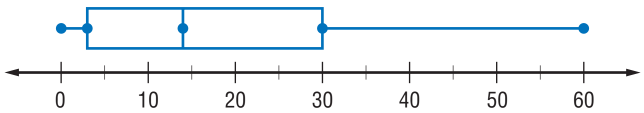
b. What was the greatest amount of time José studied in a day?
The greatest value in the plot is 6, so the greatest amount of time José studied in a day was 6 hours.

c. What is the interquartile range of this box-and-whisker plot?
The interquartile range is UQ − LQ. For this plot, the interquartile range is 3.25 − 1.5 or 1.75 hours.

d. Identify any outliers in the data.
An outlier is at least 1.5(1.75) less than the lower quartile or more than the upper quartile. Since $3.25 + (1.5)(1.75) = 5.875$, and $6 > 5.875$, the value 6 is an outlier, and was not included in the whisker.

Exercises

DRIVING **For Exercises 1–3, use the following information.**
Tyler surveyed 20 randomly chosen students at his school about how many miles they drive in an average day. The results are shown in the box-and-whisker plot.



1. What percent of the students drive more than 30 miles in a day?
2. What is the interquartile range of the box-and-whisker plot?
3. Does a student at Tyler's school have a better chance to meet someone who drives the same mileage they do if they drive 50 miles in a day or 15 miles in a day? Why?
4. **SOFT DRINKS** Carlos surveyed his friends to find the number of cans of soft drink they drink in an average week. Make a box-and-whisker plot of the data.
   {0, 0, 0, 1, 1, 1, 2, 2, 3, 4, 4, 5, 5, 7, 10, 10, 10, 11, 11}
5. **ANIMALS** The average life span of some animals commonly found in a zoo are given below. Make a box-and-whisker plot of the data.
   {1, 7, 7, 10, 12, 12, 15, 15, 18, 20, 20, 20, 25, 40, 100}
6. **BASEBALL** The table shows the number of sacrifice hits made by teams in the National Baseball League in the 2001 season. Make a box-and-whisker plot of the data.

Team	Home Runs	Team	Home Runs
Arizona	71	Milwaukee	65
Atlanta	64	Montreal	64
Chicago	117	New York	52
Cincinnati	66	Philadelphia	67
Colorado	81	Pittsburgh	60
Florida	60	San Diego	29
Houston	71	San Francisco	67
Los Angeles	57	St. Louis	83

Source: ESPN

Extra Practice

Lesson 1-1

(pages 6–10)

Find the value of each expression.

1. $2(3 + 8) - 3$
2. $(5 + 3) - 16 \div 4$
3. $4 + 8(4) \div 2 - 10$
4. $15 \div 3 \cdot 5 + 1$
5. $3(2^2 + 3)$
6. $5 + 3^2 - 16 + 4$
7. $[(4 + 8)^2 \div 9] \cdot 5$
8. $5 + 8^2 \div 4 \cdot 3$
9. $5 \cdot 7 - 2(5 + 1) \div 3$
10. $3 + 7^2 - 16 \div 2$
11. $12 + 20 \div 4 - 5$
12. $0.5[7 - (8 - 6)^2] - 1$
13. $\frac{1}{2}(3^2 + 5 \cdot 7) - 8$
14. $\frac{3 \cdot 5 + 3^2}{2^3}$
15. $\frac{6^2 + 4(2^4)}{28 + 9 \cdot 8}$
16. $\frac{2^3 - 8(4^2)}{18 + 2^5}$

Evaluate each expression if $a = -0.5$, $b = 4$, $c = 5$, and $d = -3$.

17. $3b + 4d$
18. $ab^2 + c$
19. $bc + d \div a$
20. $7ab - 3d$
21. $ad + b^2 - c$
22. $\frac{4a + 3c}{3b}$
23. $\frac{3ab^2 - d^3}{a}$
24. $\frac{5a + ad}{bc}$

Lesson 1-2

(pages 11–18)

Name the sets of numbers to which each number belongs. (Use N, W, Z, Q, I, and R.)

1. 8.2
2. -9
3. $\sqrt{36}$
4. $-\frac{1}{3}$
5. $\sqrt{2}$
6. $-0.\overline{24}$

Name the property illustrated by each equation.

7. $(4 + 9a)2b = 2b(4 + 9a)$
8. $3\left(\frac{1}{3}\right) = 1$
9. $a(3 - 2) = a \cdot 3 - a \cdot 2$
10. $(-3b) + 3b = 0$
11. $jk + 0 = jk$
12. $(2a)b = 2(ab)$

Name the additive inverse and multiplicative inverse for each number.

13. 3
14. $-\frac{1}{8}$
15. 0.2
16. $-2\frac{2}{7}$

Simplify each expression.

17. $7s + 9t + 2s - 7t$
18. $6(2a + 3b) + 5(3a - 4b)$
19. $4(3x - 5y) - 8(2x + y)$
20. $0.2(5m - 8) + 0.3(6 - 2m)$
21. $\frac{1}{2}(7p + 3q) + \frac{3}{4}(6p - 4q)$
22. $\frac{4}{5}(3v - 2w) - \frac{1}{5}(7v - 2w)$

Lesson 1-3

(pages 20–27)

Write an algebraic expression to represent each verbal expression.

1. twelve decreased by the square of a number
2. twice the sum of a number and negative nine
3. the product of the square of a number and 6
4. the square of the sum of a number and 11

Name the property illustrated by each statement.

5. If $a + 1 = 6$, then $3(a + 1) = 3(6)$
6. If $x + (4 + 5) = 21$, then $x + 9 = 21$
7. If $7x = 42$, then $7x - 5 = 42 - 5$
8. If $3 + 5 = 8$ and $8 = 2 \cdot 4$, then $3 + 5 = 2 \cdot 4$.

Solve each equation.

9. $5t + 8 = 88$
10. $27 - x = -4$
11. $\frac{3}{4}y = \frac{2}{3}y + 5$
12. $8s - 3 = 5(2s + 1)$
13. $3(k - 2) = k + 4$
14. $0.5z + 10 = z + 4$
15. $8q - \frac{q}{3} = 46$
16. $-\frac{2}{7}r + \frac{3}{7} = 5$
17. $d - 1 = \frac{1}{2}(d - 2)$

Solve each equation or formula for the specified variable.

18. $C = \pi r$; for r
19. $I = Prt$, for t
20. $m = \frac{n - 2}{n}$, for n

Lesson 1-4

(pages 28–32)

Evaluate each expression if $x = -5$, $y = 3$, and $z = -2.5$.

1. $|2x|$
2. $|-3y|$
3. $|2x + y|$
4. $|y + 5z|$
5. $-|x + z|$
6. $8 - |5y - 3|$
7. $2|x| - 4|2 + y|$
8. $|x + y| - 6|z|$

Solve each equation.

9. $|d + 1| = 7$
10. $|a - 6| = 10$
11. $2|x - 5| = 22$
12. $|t + 9| - 8 = 5$
13. $|p + 1| + 10 = 5$
14. $6|g - 3| = 42$
15. $2|y + 4| = 14$
16. $|3b - 10| = 2b$
17. $|3x + 7| + 4 = 0$
18. $|2c + 3| - 15 = 0$
19. $7 - |m - 1| = 3$
20. $3 + |z + 5| = 10$
21. $4|h + 1| = 32$
22. $2|2x + 3| = 34$
23. $3|a - 5| - 4 = 14$
24. $2|2d - 7| + 1 = 35$
25. $|3t + 6| + 9 = 30$
26. $|d - 3| = 2d + 9$
27. $|4y - 5| + 4 = 7y + 8$
28. $|2b + 4| - 3 = 6b + 1$
29. $|5t| + 2 = 3t + 18$

Lesson 1-5

(pages 33–39)

Solve each inequality. Describe the solution set using set-builder or interval notation. Then, graph the solution set on a number line.

1. $2z + 5 \leq 7$
2. $3r - 8 > 7$
3. $0.75b < 3$
4. $-3x > 6$
5. $2(3f + 5) \geq 28$
6. $-33 > 5g + 7$
7. $-3(y - 2) \geq -9$
8. $7a + 5 > 4a - 7$
9. $5(b - 3) \leq b - 7$
10. $3(2x - 5) < 5(x - 4)$
11. $2(4m - 1) + 3(m + 4) \geq 6m$
12. $8(2c - 1) > 11c + 22$
13. $5y - 4(2y + 1) \leq 2(0.5 - 2y)$
14. $2(d + 4) - 5 \geq 5(d + 3)$
15. $8 - 3t < 4(3 - t)$
16. $-x \geq \frac{x + 4}{7}$
17. $\frac{a + 8}{4} \leq \frac{7 + a}{3}$
18. $-y < \frac{y + 5}{2}$
19. $2 + 4(d - 2) \leq 3 - (d - 1)$
20. $5(x - 1) - 4x \geq 3(3 - x)$
21. $6s - (4s + 7) > 5 - s$

Define a variable and write an inequality for each problem. Then solve the resulting inequality.

22. The product of 7 and a number is greater than 42.
23. The difference of twice a number and 3 is at most 11.
24. The product of −10 and a number is greater than or equal to 20.
25. Thirty increased by a number is less than twice the number plus three.

Lesson 1-6

(pages 40–46)

Write an absolute value inequality for each of the following. Then graph the solution set on a number line.

1. all numbers less than −9 and greater than 9
2. all numbers between −5.5 and 5.5
3. all numbers greater than or equal to −2 and less than or equal to 2

Solve each inequality. Graph the solution set on a number line.

4. $3m - 2 < 7$ or $2m + 1 > 13$
5. $2 < n + 4 < 7$
6. $-3 \leq s - 2 \leq 5$
7. $5t + 3 \leq -7$ or $5t - 2 \geq 8$
8. $7 \leq 4x + 3 \leq 19$
9. $4x + 7 < 5$ or $2x - 4 > 12$
10. $|7x| \geq 21$
11. $|8p| \leq 16$
12. $|7d| \geq -42$
13. $|a + 3| < 1$
14. $|t - 4| > 1$
15. $|2y - 5| < 3$
16. $|3d + 6| \geq 3$
17. $|4x - 1| < 5$
18. $|6v + 12| > 18$
19. $|2r + 4| < 6$
20. $|5w - 3| \geq 9$
21. $|z + 2| \geq 0$
22. $12 + |2q| < 0$
23. $|3h| + 15 < 0$
24. $|5n| - 16 \geq 4$

Lesson 2-1
(pages 56–62)

Determine whether each relation is a function. Write *yes* or *no*.

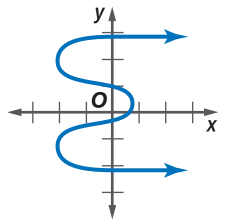
1.

Year	Population
1970	11,605
1980	13,468
1990	15,630
2000	18,140

2.

x	y
1	5
2	5
3	5
4	5

3.



Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function.

4. $\{(1, 2), (2, 3), (3, 4), (4, 5)\}$
5. $\{(0, 3), (0, 2), (0, 1), (0, 0)\}$
6. $y = -x$
7. $y = 2x - 1$
8. $y = 2x^2$
9. $y = -x^2$

Find each value if $f(x) = x + 7$ and $g(x) = (x + 1)^2$.

10. $f(2)$
11. $f(-4)$
12. $f(a + 2)$
13. $g(4)$
14. $g(-2)$
15. $f(0.5)$
16. $g(b - 1)$
17. $g(3c)$

Lesson 2-2
(pages 63–67)

State whether each equation or function is linear. Write *yes* or *no*. If no, explain.

1. $\frac{x}{2} - y = 7$
2. $\sqrt{x} = y + 5$
3. $g(x) = \frac{2}{x - 3}$
4. $x = 3 + y$
5. $f(x) = 7$
6. $\frac{3}{x} - \frac{1}{4} = \frac{4}{3}$

Write each equation in standard form. Identify *A*, *B*, and *C*.

7. $x + 7 = y$
8. $x = -3y$
9. $5x = 7y + 3$
10. $y = \frac{2}{3}x + 8$
11. $-0.4x = 10$
12. $0.75y = -6$

Find the *x*-intercept and the *y*-intercept of the graph of each equation. Then graph the equation.

13. $2x + y = 6$
14. $3x - 2y = -12$
15. $y = -x$
16. $x = 3y$
17. $\frac{3}{4}y - x = 1$
18. $y = -3$

Lesson 2-3
(pages 68–74)

Find the slope of the line that passes through each pair of points.

1. $(0, 3), (5, 0)$
2. $(2, 3), (5, 7)$
3. $(2, 8), (2, -8)$
4. $(1.5, -1), (3, 1.5)$
5. $\left(-\frac{1}{2}, \frac{3}{5}\right), \left(\frac{3}{10}, -\frac{1}{4}\right)$
6. $(-3, c), (4, c)$

Graph the line passing through the given point with the given slope.

7. $(0, 3); 1$
8. $(2, 3); 0$
9. $(-1, 1); -\frac{1}{3}$

Graph the line that satisfies each set of conditions.

10. passes through $(0, 1)$, parallel to a line whose slope is -2
11. passes through $(2, -3)$, perpendicular to a line whose slope is 5
12. passes through $(1, -1)$, parallel to the graph of $x + y = 3$
13. passes through $(4, -5)$, perpendicular to the graph of $-2x + 5y = 1$

Lesson 2-4

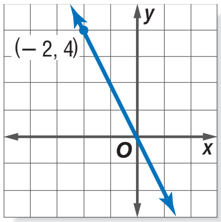
(pages 75–80)

State the slope and y-intercept of the graph of each equation.

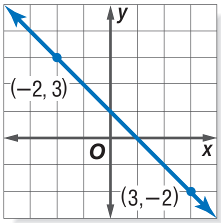
1. $y = -3x + 4$ **2.** $x - y = 5$ **3.** $x = -4$

Write an equation in slope-intercept form for each graph.

4.



5.



Write an equation in slope-intercept form for the line that satisfies each set of conditions.

6. slope -1, passes through $(7, 2)$

7. slope $\frac{3}{4}$, passes through the origin

8. passes through $(1, -3)$ and $(-1, 2)$

9. x-intercept -5, y-intercept 2

10. passes through $(1, 1)$, parallel to the graph of $2x + 3y = 5$

11. passes through $(6, -2)$, perpendicular to the graph of $-x + 8y = 5$

12. passes through $(0, 0)$, perpendicular to the graph of $2y + 3x = 4$

Lesson 2-5

(pages 81–86)

Complete parts a–c for each set of data in Exercises 1–3.

a. Draw a scatter plot.

b. Use two ordered pairs to write a prediction equation.

c. Use your prediction equation to predict the missing value.

1.

Telephone Costs	
Minutes	Cost ($)
1	0.20
3	0.52
4	0.68
6	1.00
9	1.48
15	?

2.

Washington	
Year	Population
1960	2,853,214
1970	3,413,244
1980	4,132,353
1990	4,866,669
2000	5,894,121
2010	?

Source: *The World Almanac*

3.

Federal Minimum Wage	
Year	Wage
1981	$3.35
1990	$3.80
1991	$4.25
1996	$4.75
1997	$5.15
2005	?

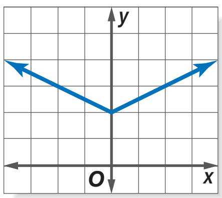
Source: *The World Almanac*

Lesson 2-6

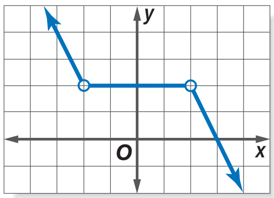
(pages 89–95)

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

1.



2.



Graph each function. Identify the domain and range.

3. $f(x) = [\![x + 5]\!]$ **4.** $g(x) = [\![x]\!] - 2$ **5.** $f(x) = -2[\![x]\!]$ **6.** $h(x) = |x| - 3$

7. $h(x) = |x - 1|$ **8.** $g(x) = |2x| + 2$ **9.** $h(x) = \begin{cases} x \text{ if } x < -2 \\ 4 \text{ if } x \geq -2 \end{cases}$ **10.** $f(x) = \begin{cases} -3 \text{ if } x \leq 1 \\ -x \text{ if } x > 1 \end{cases}$

Lesson 2-7

(pages 96–99)

Graph each inequality.

1. $y \geq x - 2$
2. $y < -3x - 1$
3. $4y \leq -3x + 8$
4. $3x > y$
5. $x + 2 \geq y - 7$
6. $2x < 5 - y$
7. $y > \frac{1}{5}x - 8$
8. $2y - 5x \leq 8$
9. $-2x + 5 \leq \frac{2}{3}y$
10. $3x + 2y \geq 0$
11. $x \leq 2$
12. $\frac{y}{2} \leq x - 1$
13. $y - 3 < 5$
14. $y \geq -|x|$
15. $|x| \leq y + 3$
16. $y > |5x - 3|$
17. $y \leq |8 - x|$
18. $y < |x + 3| - 1$
19. $y + |2x| \geq 4$
20. $y \geq |2x - 1| + 5$
21. $y < \left|\frac{2x}{3}\right| - 1$

Lesson 3-1

(pages 110–115)

Solve each system of equations by graphing.

1. $x + 3y = 18$; $-x + 2y = 7$
2. $x - y = 2$; $2x - 2y = 10$
3. $2x + 6y = 6$; $\frac{1}{3}x + y = 1$
4. $x + 3y = 0$; $2x + 6y = 5$
5. $2x - y = 7$; $\frac{2}{5}x - \frac{4}{3}y = -2$
6. $y = \frac{1}{3}x + 1$; $y = 4x + 1$

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

7. $2x + 3y = 5$; $-6x - 9y = -15$
8. $x - 2y = 4$; $y = x - 2$
9. $y = 0.5x$; $2y = x + 4$
10. $9x - 5 = 7y$; $4.5x - 3.5y = 2.5$
11. $\frac{3}{4}x - y = 0$; $\frac{1}{3}y + \frac{1}{2}x = 6$
12. $\frac{2}{3}x = \frac{5}{3}y$; $2x - 5y = 0$

Lesson 3-2

(pages 116–122)

Solve each system of equations by using substitution.

1. $2x + 3y = 10$; $x + 6y = 32$
2. $x = 4y - 10$; $5x + 3y = -4$
3. $3x - 4y = -27$; $2x + y = -7$

Solve each system of equations by using elimination.

4. $7x + y = 9$; $5x - y = 15$
5. $r + 5s = -17$; $2r - 6s = -2$
6. $6p + 8q = 20$; $5p - 4q = -26$

Solve each system of equations by using either substitution or elimination.

7. $2x - 3y = 7$; $3x + 6y = 42$
8. $2a + 5b = -13$; $3a - 4b = 38$
9. $3c + 4d = -1$; $6c - 2d = 3$
10. $7x - y = 35$; $y = 5x - 19$
11. $3m + 4n = 28$; $5m - 3n = -21$
12. $x = 2y - 1$; $4x - 3y = 21$
13. $2.5x + 1.5y = -2$; $3.5x - 0.5y = 18$
14. $\frac{5}{2}x + \frac{1}{3}y = 13$; $\frac{1}{2}x - y = -7$
15. $\frac{2}{7}c - \frac{4}{3}d = 16$; $\frac{4}{7}c + \frac{8}{3}d = -16$

Lesson 3-3

(pages 123–127)

Solve each system of inequalities by graphing.

1. $x \leq 5$
   $y \geq -3$
2. $y < 3$
   $y - x \geq -1$
3. $x + y < 5$
   $x < 2$
4. $y + x < 2$
   $y \geq x$
5. $x + y \leq 2$
   $y - x \leq 4$
6. $y \leq x + 4$
   $y - x \geq 1$
7. $y < \frac{1}{3}x + 5$
   $y > 2x + 1$
8. $y + x \geq 1$
   $y - x \geq -1$
9. $|x| > 2$
   $|y| \leq 5$
10. $|x - 3| \leq 3$
    $4y - 2x \leq 6$
11. $4x + 3y \geq 12$
    $2y - x \geq -1$
12. $y \leq -1$
    $3x - 2y \geq 6$

Find the coordinates of the vertices of the figure formed by each system of inequalities.

13. $y \leq 3$
    $x \leq 2$
    $y \geq -\frac{3}{2}x + 3$
14. $y \geq -1$
    $y \leq x$
    $y \leq -x + 4$
15. $y \leq \frac{1}{3}x + \frac{7}{3}$
    $4x - y \leq 5$
    $y \geq -\frac{3}{2}x + \frac{1}{2}$

Lesson 3-4

(pages 129–135)

A feasible region has vertices at (−3, 2), (1, 3), (6, 1), and (2, −2). Find the maximum and minimum values of each function.

1. $f(x, y) = 2x - y$
2. $f(x, y) = x + 5y$
3. $f(x, y) = y - 4x$
4. $f(x, y) = -x + 3y$
5. $f(x, y) = 3x - y$
6. $f(x, y) = 2y - 2x$

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

7. $4x - 5y \leq -10$
   $y \leq 6$
   $2x + y \geq 2$
   $f(x, y) = x + y$
8. $x \leq 5$
   $y \geq 2$
   $2x - 5y \geq -10$
   $f(x, y) = 3x + y$
9. $x - 2y \geq -7$
   $x + y \leq 8$
   $y \geq 5x + 8$
   $f(x, y) = 3x - 4y$
10. $y \leq 4x + 6$
    $x + 4y \geq 7$
    $2x + y \leq 7$
    $f(x, y) = 2x - y$
11. $y \geq 0$
    $y \leq 5$
    $y \leq -x + 7$
    $5x + 3y \geq 20$
    $f(x, y) = x + 2y$
12. $y \geq 0$
    $3x - 2y \geq 0$
    $x + 3y \leq 11$
    $2x + 3y \leq 16$
    $f(x, y) = 4x + y$

Lesson 3-5

(pages 138–144)

For each system of equations, an ordered triple is given. Determine whether or not it is a solution of the system.

1. $4x + 2y - 6z = -38$
   $5x - 4y + z = -18$
   $x + 3y + 7z = 38$; $(-3, 2, 5)$
2. $u + 3v + w = 14$
   $2u - v + 3w = -9$
   $4u - 5v - 2w = -2$; $(1, 5, -2)$
3. $x + y = -6$
   $x + z = -2$
   $y + z = 2$; $(-4, -2, 2)$

Solve each system of equations.

4. $5a = 5$
   $6b - 3c = 15$
   $2a + 7c = -5$
5. $s + 2t = 5$
   $7r - 3s + t = 20$
   $2t = 8$
6. $2u - 3v = 13$
   $3v + w = -3$
   $4u - w = 2$
7. $4a + 2b - c = 5$
   $2a + b - 5c = -11$
   $a - 2b + 3c = 6$
8. $x + 2y - z = 1$
   $x + 3y + 2z = 7$
   $2x + 6y + z = 8$
9. $2x + y - z = 7$
   $3x - y + 2z = 15$
   $x - 4y + z = 2$

Extra Practice

Lesson 4-1

(pages 154–158)

State the dimensions of each matrix.

1. $[8 \quad 4 \quad 3]$

2. $\begin{bmatrix} 2 & -1 \\ 0 & 6 \end{bmatrix}$

3. $\begin{bmatrix} 4 \\ -3 \end{bmatrix}$

Solve each matrix equation.

4. $[2x \quad 3y \quad -z] = [2y \quad -z \quad 15]$

5. $\begin{bmatrix} x + y \\ 4x - 3y \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \end{bmatrix}$

6. $-2\begin{bmatrix} w + 5 & x - z \\ 3y & 8 \end{bmatrix} = \begin{bmatrix} -16 & -4 \\ 6 & 2x + 8z \end{bmatrix}$

7. $y\begin{bmatrix} 2 & x \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -10 \\ 10 & 2z \end{bmatrix}$

8. $\begin{bmatrix} 2x \\ -y \\ 3z \end{bmatrix} = \begin{bmatrix} 16 \\ 18 \\ -21 \end{bmatrix}$

9. $\begin{bmatrix} x - 3y \\ 4y - 3x \end{bmatrix} = -5\begin{bmatrix} 2 \\ x \end{bmatrix}$

10. $\begin{bmatrix} x^2 + 4 & y + 6 \\ x - y & 2 - y \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 0 & 1 \end{bmatrix}$

11. $\begin{bmatrix} x + y & 3 \\ y & 6 \end{bmatrix} = \begin{bmatrix} 0 & 2y - x \\ z & 4 - 2x \end{bmatrix}$

Lesson 4-2

(pages 160–166)

Perform the indicated matrix operations. If the matrix does not exist, write impossible.

1. $\begin{bmatrix} 3 & 5 \\ -7 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 6 \\ 8 & -1 \end{bmatrix}$

2. $[0 \quad -1 \quad 3] + \begin{bmatrix} 5 \\ -2 \\ -3 \end{bmatrix}$

3. $\begin{bmatrix} 45 & 36 & 18 \\ 63 & 29 & 5 \end{bmatrix} - \begin{bmatrix} 45 & -2 & 36 \\ 18 & 9 & -10 \end{bmatrix}$

4. $4[-8 \quad 2 \quad 9] - 3[2 \quad -7 \quad 6]$

5. $\begin{bmatrix} -3 & 6 & -9 \\ 4 & -3 & 0 \\ 8 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 7 \\ 5 & 2 & -6 \\ 3 & 0 & -2 \end{bmatrix}$

6. $5\begin{bmatrix} 3 & 1 & 0 \\ 0 & 0 & 2 \\ 1 & -1 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 3 \\ 1 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$

7. $5\begin{bmatrix} 6 & -2 \\ 5 & 4 \end{bmatrix} - 2\begin{bmatrix} 6 & -2 \\ 5 & 4 \end{bmatrix} + 4\begin{bmatrix} 7 & -6 \\ -4 & 2 \end{bmatrix}$

8. $1.3\begin{bmatrix} 3.7 \\ -5.4 \end{bmatrix} + 4.1\begin{bmatrix} 6.4 \\ -3.7 \end{bmatrix} - 6.2\begin{bmatrix} -0.8 \\ 7.4 \end{bmatrix}$

Use the matrices A, B, C, D, and E to find the following.

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & -2 \\ -3 & 3 \end{bmatrix}$, $D = \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}$, $E = \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix}$

9. $A + B$
10. $C + D$
11. $A - B$
12. $4B$
13. $D - C$
14. $E + 2A$
15. $D - 2B$
16. $2A + 3E - D$

Lesson 4-3

(pages 167–174)

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1. $A_{4 \times 3} \cdot B_{3 \times 4}$
2. $R_{2 \times 5} \cdot S_{2 \times 5}$
3. $G_{2 \times 5} \cdot H_{2 \times 3}$
4. $M_{3 \times 8} \cdot N_{8 \times 2}$
5. $X_{4 \times 3} \cdot Y_{2 \times 6}$
6. $J_{5 \times 3} \cdot K_{3 \times 6}$
7. $C_{m \times n} \cdot D_{n \times p}$
8. $X_{2 \times 9} \cdot Y_{9 \times 7}$

Find each product, if possible.

9. $[-3 \quad 4] \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

10. $\begin{bmatrix} 2 & -4 \\ 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ -2 & -1 \end{bmatrix}$

11. $\begin{bmatrix} 1 & 3 \\ -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -4 \\ 0 & 5 \end{bmatrix}$

12. $\begin{bmatrix} 3 & 2 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} -8 \\ 15 \end{bmatrix}$

13. $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 7 & 6 & 1 \\ 2 & -4 & 0 \end{bmatrix}$

14. $\begin{bmatrix} 0 & 1 & -2 \\ 5 & 3 & -4 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & -2 \end{bmatrix}$

15. $\begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

16. $\begin{bmatrix} -1 & 0 & 2 \\ -6 & 5 & -3 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$

Lesson 4-4

(pages 175–181)

For Exercises 1–3, use the following information.
Triangle XYZ with vertices $X(2, 5)$, $Y(-3, 1)$, and $Z(1, -4)$ is translated 3 units right and 2 units down.

1. Write the translation matrix.
2. Find the coordinates of $\triangle X'Y'Z'$.
3. Graph the preimage and the image.

For Exercises 4–6, use the following information.
The vertices of quadrilateral $ABCD$ are $A(1, 1)$, $B(-2, 3)$, $C(-4, -1)$, and $D(2, -3)$. The quadrilateral is dilated so that its perimeter is 2 times the original perimeter.

4. Write the coordinates for $ABCD$ in a vertex matrix.
5. Find the coordinates of the image $A'B'C'D'$.
6. Graph $ABCD$ and $A'B'C'D'$.

For Exercises 7–13, use the following information.
The vertices of $\triangle MQN$ are $M(2, 4)$, $Q(3, -5)$, and $N(1, -1)$.

7. Write the coordinates of $\triangle MQN$ in a vertex matrix.
8. Write the reflection matrix for reflecting over the line $y = x$.
9. Find the coordinates of $\triangle M'Q'N'$ after the reflection.
10. Graph $\triangle MQN$ and $\triangle M'Q'N'$.
11. Write a rotation matrix for rotating $\triangle MQN$ 90° counterclockwise about the origin.
12. Find the coordinates of $\triangle M'Q'N'$ after the rotation.
13. Graph $\triangle MQN$ and $\triangle M'Q'N'$.

Lesson 4-5

(pages 182–188)

Find the value of each determinant.

1. $\begin{vmatrix} 1 & -5 \\ 3 & 4 \end{vmatrix}$
2. $\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$
3. $\begin{vmatrix} 2 & -2 \\ 3 & -3 \end{vmatrix}$
4. $\begin{vmatrix} 2 & -3 \\ -3 & -2 \end{vmatrix}$

Evaluate each determinant using expansion by minors.

5. $\begin{vmatrix} 2 & -3 & 5 \\ 1 & -2 & -7 \\ -1 & 4 & -3 \end{vmatrix}$
6. $\begin{vmatrix} 0 & -1 & 2 \\ -2 & 1 & 0 \\ 2 & 0 & -1 \end{vmatrix}$
7. $\begin{vmatrix} 4 & 3 & -2 \\ 2 & 5 & -8 \\ 6 & 4 & -1 \end{vmatrix}$
8. $\begin{vmatrix} -3 & 0 & 2 \\ 1 & -2 & -1 \\ 0 & 5 & 0 \end{vmatrix}$

Evaluate each determinant using diagonals.

9. $\begin{vmatrix} 3 & 2 & -1 \\ 2 & 3 & 0 \\ -1 & 0 & 3 \end{vmatrix}$
10. $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$
11. $\begin{vmatrix} 6 & 4 & -1 \\ 2 & 5 & -8 \\ 4 & 3 & -2 \end{vmatrix}$
12. $\begin{vmatrix} 6 & 12 & 15 \\ 9 & 3 & 14 \\ 5 & 6 & 3 \end{vmatrix}$

Lesson 4-6

(pages 189–194)

Use Cramer's Rule to solve each system of equations.

1. $5x - 3y = 19$
   $7x + 2y = 8$
2. $4p - 3q = 22$
   $2p + 8q = 30$
3. $-x + y = 5$
   $2x + 4y = 38$
4. $\frac{1}{3}x - \frac{1}{2}y = -8$
   $\frac{3}{5}x + \frac{5}{6}y = -4$
5. $\frac{1}{4}c + \frac{2}{3}d = 6$
   $\frac{3}{4}c - \frac{5}{3}d = -4$
6. $0.3a + 1.6b = 0.44$
   $0.4a + 2.5b = 0.66$
7. $x + y + z = 6$
   $2x - y - z = -3$
   $3x + y - 2z = -1$
8. $2a + b - c = -6$
   $a - 2b + c = 8$
   $-a - 3b + 2c = 14$
9. $r + 2s - t = 10$
   $-2r + 3s + t = 6$
   $3r - 2s + 2t = -19$

Lesson 4-7

(pages 195–201)

Determine whether each pair of matrices are inverses.

1. $A = \begin{bmatrix} -7 & -6 \\ 8 & 7 \end{bmatrix}, B = \begin{bmatrix} -7 & -6 \\ 8 & 7 \end{bmatrix}$

2. $C = \begin{bmatrix} -3 & 4 \\ 2 & -2 \end{bmatrix}, D = \begin{bmatrix} -2 & -2 \\ -4 & -3 \end{bmatrix}$

3. $X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

4. $N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, M = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Find the inverse of each matrix, if it exists.

5. $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

6. $\begin{bmatrix} 3 & 2 \\ 0 & -4 \end{bmatrix}$

7. $\begin{bmatrix} 3 & 8 \\ 0 & -1 \end{bmatrix}$

8. $\begin{bmatrix} 3 & -6 \\ 2 & -4 \end{bmatrix}$

9. $\begin{bmatrix} 2 & 4 \\ 2 & 3 \end{bmatrix}$

10. $\begin{bmatrix} 8 & -5 \\ -6 & 4 \end{bmatrix}$

11. $\begin{bmatrix} 10 & 3 \\ 5 & -2 \end{bmatrix}$

12. $\begin{bmatrix} -3 & 4 \\ -4 & 8 \end{bmatrix}$

13. $\begin{bmatrix} -1 & -3 \\ -2 & -4 \end{bmatrix}$

14. $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

15. $\begin{bmatrix} 3 & -3 \\ 3 & -3 \end{bmatrix}$

16. $\begin{bmatrix} \frac{1}{3} & -\frac{1}{2} \\ -\frac{1}{4} & \frac{1}{5} \end{bmatrix}$

Lesson 4-8

(pages 202–207)

Write a matrix equation for each system of equations.

1. $5a + 3b = 6$
$2a - b = 9$

2. $3x + 4y = -8$
$2x - 3y = 6$

3. $m + 3n = 1$
$4m - n = -22$

4. $4c - 3d = -1$
$5c - 2d = 39$

5. $x + 2y - z = 6$
$-2x + 3y + z = 1$
$x + y + 3z = 8$

6. $2a - 3b - c = 4$
$4a + b + c = 15$
$a - b - c = -2$

Solve each matrix equation or system of equations by using inverse matrices.

7. $\begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 33 \\ -1 \end{bmatrix}$

8. $\begin{bmatrix} -1 & 1 \\ 7 & -6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -29 \\ 52 \end{bmatrix}$

10. $5x - y = 7$
$8x + 2y = 4$

11. $3m + n = 4$
$2m + 2n = 3$

12. $6c + 5d = 7$
$3c - 10d = -4$

13. $3a - 5b = 1$
$a + 3b = 5$

14. $2r - 7s = 24$
$-r + 8s = -21$

15. $x + y = -3$
$3x - 10y = 43$

16. $2m - 3n = 3$
$-4m + 9n = -8$

17. $x + y = 1$
$2x - 2y = -12$

Lesson 5-1

(pages 222–228)

Simplify. Assume that no variable equals 0.

1. $x^7 \cdot x^3 \cdot x$

2. $m^8 \cdot m \cdot m^{10}$

3. $7^5 \cdot 7^2$

4. $(-3)^4(-3)$

5. $\frac{t^{12}}{t}$

6. $-\frac{16x^8}{8x^2}$

7. $\frac{6^5}{6^3}$

8. $\frac{p^5q^7}{p^2q^5}$

9. $-(m^3)^8$

10. $\frac{x}{x^7}$

11. $(3^5)^7$

12. -3^4

13. $(abc)^3$

14. $(x^2)^5$

15. $-\left(\frac{x}{5}\right)^2$

16. $(b^4)^6$

17. $(-2y^5)^2$

18. $3x^0$

19. $(5x^4)^{-2}$

20. $\left(\frac{5a^7}{2b^5c}\right)^3$

21. $(-3)^{-2}$

22. -3^{-2}

23. $\frac{1}{x^{-3}}$

24. $\frac{5^6a^{x+y}}{5^4a^{x-y}}$

Evaluate. Express the result in scientific notation.

25. $(8.95 \times 10^9)(1.82 \times 10^7)$

26. $(3.1 \times 10^5)(7.9 \times 10^{-8})$

27. $\frac{(2.38 \times 10^{13})(7.56 \times 10^{-5})}{(4.2 \times 10^{18})}$

Lesson 5-2

(pages 229–232)

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

1. $5x - 3x^2 + 7$
2. $\sqrt{7} - u^3$
3. $5r^3 + 7r^2s - 3rs^2 + 6s^3$
4. $\frac{xy}{3}$
5. $2 + \frac{7}{w}$
6. $\sqrt{a - 1}$

Simplify.

7. $(4x^3 + 5x - 7x^2) + (-2x^3 + 5x^2 - 7y^2)$
8. $(2x^2 - 3x + 11) + (7x^2 + 2x - 8)$
9. $(-3x^2 + 7x + 23) + (-8x^2 - 5x + 13)$
10. $(-3x^2 + 7x + 23) - (-8x^2 - 5x + 13)$
11. $(5x^2 + 7x + 23) - (x^2 - 9x - 5) - (-3x^2 + 4x + 2)$
12. $5a^2b(4a - 3b)$
13. $\frac{7}{uw}\left(4u^2w^3 - 5uw + \frac{w}{7u}\right)$
14. $-4x^5(-3x^4 - x^3 + x + 7)$
15. $(2x - 3)(4x + 7)$
16. $(3x - 5)(-2x - 1)$
17. $(3x - 5)(2x - 1)$
18. $(2x + 5)(2x - 5)$
19. $(3x - 7)(3x + 7)$
20. $(5 + 2w)(5 - 2w)$
21. $(2a^2 + 8)(2a^2 - 8)$
22. $(-5x + 10)(-5x - 10)$
23. $(4x - 3)^2$
24. $(5x + 6)^2$
25. $(-x + 1)^2$
26. $\frac{3}{4}x(x^2 + 4x + 14)$
27. $-\frac{1}{2}a^2(a^3 - 6a^2 + 5a)$

Lesson 5-3

(pages 233–238)

Simplify.

1. $\frac{18r^3s^2 + 36r^2s^3}{9r^2s^2}$
2. $\frac{15v^3w^2 - 5v^4w^3}{-5v^4w^3}$
3. $\frac{x^2 - x + 1}{x}$
4. $(5bh + 5ch) \div (b + c)$
5. $(25c^4d + 10c^3d^2 - cd) \div 5cd$
6. $(16f^{18} + 20f^9 - 8f^6) \div 4f^3$
7. $(33m^5 + 55mn^5 - 11m^3)(11m)^{-1}$
8. $(8g^3 + 19g^2 - 12g + 9) \div (g + 3)$
9. $(p^{21} + 3p^{14} + p^7 - 2)(p^7 + 2)^{-1}$
10. $(15x^3 + 19x^2y - 7xy^2 + 5y^3) \div (3x + 5y)$
11. $(8k^2 - 56k + 98) \div (2k - 7)$
12. $(2r^2 + 5r - 3) \div (r + 3)$
13. $(n^3 + 125) \div (n + 5)$

Use synthetic division to find each quotient.

14. $(10y^4 + 3y^2 - 7) \div (2y^2 - 1)$
15. $(q^4 + 8q^3 + 3q + 17) \div (q + 8)$
16. $(15v^3 + 8v^2 - 21v + 6) \div (5v - 4)$
17. $(-2x^3 + 15x^2 - 10x + 3) \div (x + 3)$
18. $(6a^4 - 22a^3 - 9a^2 + 9a - 17) \div (a - 4)$
19. $(5s^3 + s^2 - 7) \div (s + 1)$
20. $(t^4 - 2t^3 + t^2 - 3t + 2) \div (t - 2)$
21. $(z^4 - 3z^3 - z^2 - 11z - 4) \div (z - 4)$
22. $(3r^4 - 6r^3 - 2r^2 + r - 6) \div (r + 1)$
23. $(2b^3 - 11b^2 + 12b + 9) \div (b - 3)$

Lesson 5-4

(pages 239–244)

Factor completely. If the polynomial is not factorable, write *prime*.

1. $14a^3b^3c - 21a^2b^4c + 7a^2b^3c$
2. $10ax - 2xy - 15ab + 3by$
3. $x^2 + x - 42$
4. $2x^2 + 5x + 3$
5. $6x^2 + 71x - 12$
6. $6x^4 - 12x^3 + 3x^2$
7. $x^2 - 6x + 2$
8. $x^2 - 2x - 15$
9. $6x^2 + 23x + 20$
10. $24x^2 - 76x + 40$
11. $6p^2 - 13pq - 28q^2$
12. $2x^2 - 6x + 3$
13. $x^2 + 49 - 14x$
14. $9x^2 - 64$
15. $36 - t^{10}$
16. $x^2 + 16$
17. $a^4 - 81b^4$
18. $3a^3 + 12a^2 - 63a$
19. $x^3 - 8x^2 + 15x$
20. $x^2 + 6x + 9$
21. $18x^3 - 8x$
22. $3x^2 - 42x + 40$
23. $2x^2 + 4x - 1$
24. $2x^3 + 6x^2 + x + 3$
25. $35ac - 3bd - 7ad + 15bc$
26. $5h^2 - 10hj + h - 2j$

Simplify. Assume that no denominator is equal to 0.

27. $\frac{x^2 + 8x + 15}{x^2 + 4x + 3}$
28. $\frac{x^2 + x - 2}{x^2 - 6x + 5}$
29. $\frac{x^2 - 15x + 56}{x^2 - 4x - 21}$
30. $\frac{x^2 + x - 6}{x^3 + 9x^2 + 27x + 27}$

Extra Practice

Lesson 5-5

(pages 245–249)

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{289}$ 2. $\sqrt{7832}$ 3. $\sqrt[4]{0.0625}$ 4. $\sqrt[3]{-343}$

5. $\sqrt[10]{32^4}$ 6. $\sqrt[3]{49}$ 7. $\sqrt[5]{5}$ 8. $-\sqrt[4]{25}$

Simplify.

9. $\sqrt{9h^{22}}$ 10. $\sqrt[5]{0}$ 11. $\sqrt{\frac{16}{9}}$ 12. $\sqrt{\left(-\frac{2}{3}\right)^4}$

13. $\sqrt[5]{-32}$ 14. $-\sqrt{-144}$ 15. $\sqrt[4]{a^{16}b^8}$ 16. $\pm\sqrt[4]{81x^4}$

17. $\sqrt[5]{\frac{1}{100{,}000}}$ 18. $\sqrt[3]{-d^6}$ 19. $\sqrt[5]{p^{25}q^{15}r^5s^{20}}$ 20. $\sqrt[4]{(2x^2 - y^8)^8}$

21. $\pm\sqrt{16m^6n^2}$ 22. $-\sqrt[3]{(2x - y)^3}$ 23. $\sqrt[4]{(r + s)^4}$ 24. $\sqrt{9a^2 + 6a + 1}$

25. $\sqrt{4y^2 + 12y + 9}$ 26. $-\sqrt{x^2 - 2x + 1}$ 27. $\pm\sqrt{x^2 + 2x + 1}$ 28. $\sqrt[3]{a^3 + 6a^2 + 12a + 8}$

Lesson 5-6

(pages 250–256)

Simplify.

1. $\sqrt{75}$ 2. $7\sqrt{12}$ 3. $\sqrt[3]{81}$ 4. $\sqrt{5r^5}$

5. $\sqrt[4]{7^8x^5y^6}$ 6. $3\sqrt{5} + 6\sqrt{5}$ 7. $\sqrt{18} - \sqrt{50}$ 8. $4\sqrt[3]{32} + \sqrt[3]{500}$

9. $\sqrt{12}\,\sqrt{27}$ 10. $3\sqrt{12} + 2\sqrt{300}$ 11. $\sqrt[3]{54} - \sqrt[3]{24}$ 12. $\sqrt{10}(2 - \sqrt{5})$

13. $-\sqrt{3}(2\sqrt{6} - \sqrt{63})$ 14. $(5 + \sqrt{2})(3 + \sqrt{3})$ 15. $(2 + \sqrt{5})(2 - \sqrt{5})$ 16. $(8 + \sqrt{11})^2$

17. $(\sqrt{3} + \sqrt{6})(\sqrt{3} - \sqrt{6})$ 18. $(\sqrt{8} + \sqrt{13})^2$ 19. $(1 - \sqrt{7})(4 + \sqrt{7})$ 20. $(5 - 2\sqrt{7})^2$

21. $\sqrt{\frac{3m^3}{24n^5}}$ 22. $\frac{\sqrt{18}}{\sqrt{32}}$ 23. $2\sqrt[3]{\frac{r^5}{2s^2t}}$ 24. $\sqrt[3]{\frac{4}{7}}$

25. $\sqrt[5]{\frac{32}{a^4}}$ 26. $\sqrt{\frac{2}{3}} - \sqrt{\frac{3}{8}}$ 27. $\frac{5}{3 - \sqrt{10}}$ 28. $\frac{\sqrt{5}}{1 + \sqrt{3}}$

29. $\frac{-2 + \sqrt{7}}{2 + \sqrt{7}}$ 30. $\frac{1 - \sqrt{3}}{1 + \sqrt{8}}$ 31. $\frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}}$ 32. $\frac{x + \sqrt{5}}{x - \sqrt{5}}$

Lesson 5-7

(pages 257–262)

Write each expression in radical form.

1. $10^{\frac{1}{3}}$ 2. $8^{\frac{1}{4}}$ 3. $a^{\frac{2}{3}}$ 4. $(b^2)^{\frac{3}{4}}$

Write each radical using rational exponents.

5. $\sqrt{35}$ 6. $\sqrt[4]{32}$ 7. $\sqrt[3]{27a^2x}$ 8. $\sqrt[5]{25ab^3c^4}$

Evaluate each expression.

9. $2401^{\frac{1}{4}}$ 10. $27^{\frac{4}{3}}$ 11. $(-32)^{\frac{2}{5}}$ 12. $-81^{\frac{3}{4}}$

13. $(-125)^{-\frac{2}{3}}$ 14. $16^{\frac{5}{2}} \cdot 16^{\frac{1}{2}}$ 15. $8^{-\frac{2}{3}} \cdot 64^{\frac{1}{6}}$ 16. $\left(\frac{48}{1875}\right)^{-\frac{5}{4}}$

Simplify each expression.

17. $7^{\frac{5}{9}} \cdot 7^{\frac{4}{9}}$ 18. $32^{\frac{2}{3}} \cdot 32^{\frac{3}{5}}$ 19. $\left(k^{\frac{8}{5}}\right)^5$ 20. $x^{\frac{2}{5}} \cdot x^{\frac{8}{5}}$

21. $m^{\frac{2}{5}} \cdot m^{\frac{4}{5}}$ 22. $\left(p^{\frac{5}{4}} \cdot q^{\frac{7}{2}}\right)^{\frac{8}{3}}$ 23. $\left(4^{\frac{9}{2}}c^{\frac{3}{2}}\right)^2$ 24. $\frac{7^{\frac{3}{4}}}{7^{\frac{5}{3}}}$

25. $\frac{1}{t^{\frac{9}{5}}}$ 26. $a^{\frac{-8}{7}}$ 27. $\frac{r}{r^{\frac{7}{5}}}$ 28. $\sqrt[4]{36}$

29. $\sqrt[4]{9a^2}$ 30. $\sqrt[3]{\sqrt{81}}$ 31. $\frac{v^{\frac{11}{7}} - v^{\frac{4}{7}}}{v^{\frac{4}{7}}}$ 32. $\frac{1}{5^{\frac{1}{2}} + 3^{\frac{1}{2}}}$

Lesson 5-8

(pages 263–267)

Solve each equation or inequality.

1. $\sqrt{x} = 16$
2. $\sqrt{z+3} = 7$
3. $\sqrt[3]{a+5} = 1$
4. $5\sqrt{s} - 8 = 3$
5. $\sqrt[4]{m+7} + 11 = 9$
6. $d + \sqrt{d^2 - 8} = 4$
7. $g\sqrt{5} + 4 = g + 4$
8. $\sqrt{x-8} = \sqrt{13+x}$
9. $\sqrt{3x+10} = 1 + \sqrt{2x+5}$
10. $\sqrt{3x+9} > 2$
11. $\sqrt{3n-1} \le 5$
12. $2 - 4\sqrt{21-6c} < -6$
13. $\sqrt{5y+4} > 8$
14. $\sqrt{2w+3} + 5 \ge 7$
15. $\sqrt{x+29} - 3 = \sqrt{x-16}$
16. $\sqrt{3x+25} + \sqrt{10-2x} = 0$
17. $\sqrt{2c+3} - 7 > 0$
18. $\sqrt{3z-5} - 3 = 1$
19. $\sqrt{5y+1} + 6 < 10$
20. $\sqrt{3n+1} - 2 \le 6$
21. $\sqrt{y-5} - \sqrt{y} \ge 1$
22. $(5n-1)^{\frac{1}{2}} = 0$
23. $(7x-6)^{\frac{1}{3}} + 1 = 3$
24. $(6a-8)^{\frac{1}{4}} + 9 \ge 10$

Lesson 5-9

(pages 270–275)

Simplify.

1. $\sqrt{-289}$
2. $\sqrt{-\frac{25}{121}}$
3. $\sqrt{-625b^8}$
4. $\sqrt{-\frac{28t^6}{27s^5}}$
5. $(7i)^2$
6. $(6i)(-2i)(11i)$
7. $(\sqrt{-8})(\sqrt{-12})$
8. $-i^{22}$
9. $i^{17} \cdot i^{12} \cdot i^{26}$
10. $(14-5i) + (-8+19i)$
11. $(7i) - (2+3i)$
12. $(2+2i) - (5+i)$
13. $(7+3i)(7-3i)$
14. $(8-2i)(5+i)$
15. $(6+8i)^2$
16. $\frac{3}{6-2i}$
17. $\frac{5i}{3+4i}$
18. $\frac{3-7i}{5+4i}$

Solve each equation.

19. $x^2 + 8 = 3$
20. $\frac{4x^2}{49} + 6 = 3$
21. $8x^2 + 5 = 1$
22. $12 - 9x^2 = 38$
23. $9x^2 + 7 = 4$
24. $\frac{1}{2}x^2 + 1 = 0$

Lesson 6-1

(pages 286–293)

For Exercises 1–12, complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

1. $f(x) = 6x^2$
2. $f(x) = -x^2$
3. $f(x) = x^2 + 5$
4. $f(x) = -x^2 - 2$
5. $f(x) = 2x^2 + 1$
6. $f(x) = -3x^2 + 6x$
7. $f(x) = x^2 + 6x - 3$
8. $f(x) = x^2 - 2x - 8$
9. $f(x) = -3x^2 - 6x + 12$
10. $f(x) = x^2 + 5x - 6$
11. $f(x) = 2x^2 + 7x - 4$
12. $f(x) = -5x^2 + 10x + 1$

Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

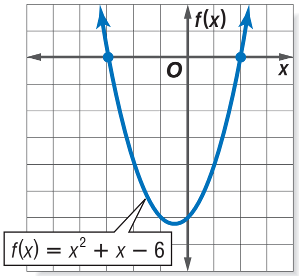
13. $f(x) = 9x^2$
14. $f(x) = 9 - x^2$
15. $f(x) = x^2 - 5x + 6$
16. $f(x) = 2 + 7x - 6x^2$
17. $f(x) = 4x^2 - 9$
18. $f(x) = x^2 + 2x + 1$
19. $f(x) = 8 - 3x - 4x^2$
20. $f(x) = x^2 - x + \frac{5}{4}$
21. $f(x) = -x^2 + \frac{14}{3}x + \frac{5}{3}$

Lesson 6-2

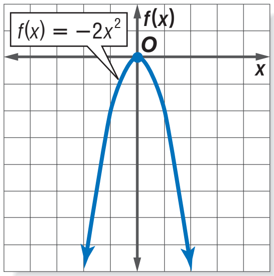
(pages 294–299)

Use the related graph of each equation to determine its solutions.

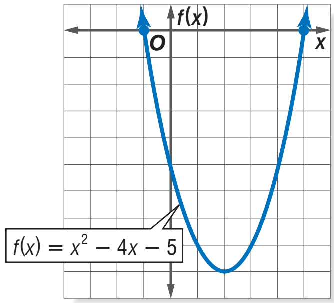
1. $x^2 + x - 6 = 0$



2. $-2x^2 = 0$



3. $x^2 - 4x - 5 = 0$



Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. $x^2 - 2x = 0$

5. $x^2 + 8x - 20 = 0$

6. $-2x^2 + 10x - 5 = 0$

7. $-5x + 2x^2 - 3 = 0$

8. $3x^2 - x + 8 = 0$

9. $-x^2 + 2 = 7x$

10. $4x^2 - 4x + 1 = 0$

11. $4x + 1 = 3x^2$

12. $x^2 = -9x$

13. $x^2 + 6x - 27 = 0$

14. $0.4x^2 + 1 = 0$

15. $0.5x^2 + 3x - 2 = 0$

Lesson 6-3

(pages 301–305)

Solve each equation by factoring.

1. $x^2 + 7x + 10 = 0$

2. $3x^2 = 75x$

3. $2x^2 + 7x = 9$

4. $8x^2 = 48 - 40x$

5. $5x^2 = 20x$

6. $12x^2 - 71x - 6 = 0$

7. $16x^2 - 64 = 0$

8. $5x^2 - 45x + 90 = 0$

9. $24x^2 - 15 = 2x$

10. $x^2 = 72 - x$

11. $2x^2 + 5x + 3 = 0$

12. $4x^2 + 9 = 12x$

13. $2x^2 - 8x = 0$

14. $8x^2 + 10x = 3$

15. $12x^2 - 5x = 3$

16. $x^2 + 8x + 12 = 0$

17. $x^2 + 9x + 14 = 0$

18. $9x^2 + 1 = 6x$

19. $6x^2 + 7x = 3$

20. $x^2 - 4x = 21$

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

21. $2, 1$

22. $-3, 4$

23. $-1, -7$

24. $-1, \frac{1}{2}$

25. $-5, \frac{1}{4}$

26. $-\frac{1}{3}, -\frac{1}{2}$

Lesson 6-4

(pages 306–312)

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

1. $x^2 - 4x + c$

2. $x^2 + 20x + c$

3. $x^2 - 11x + c$

4. $x^2 - \frac{2}{3}x + c$

5. $x^2 + 30x + c$

6. $x^2 + \frac{3}{8}x + c$

7. $x^2 - \frac{2}{5}x + c$

8. $x^2 - 3x + c$

Solve each equation by completing the square.

9. $x^2 + 3x - 4 = 0$

10. $x^2 + 5x = 0$

11. $x^2 + 2x - 63 = 0$

12. $3x^2 - 16x - 35 = 0$

13. $x^2 + 7x + 13 = 0$

14. $5x^2 - 8x + 2 = 0$

15. $x^2 - 6x + 11 = 0$

16. $x^2 - 12x + 36 = 0$

17. $8x^2 + 13x - 4 = 0$

18. $3x^2 + 5x + 6 = 0$

19. $x^2 + 14x - 1 = 0$

20. $4x^2 - 32x + 15 = 0$

21. $3x^2 - 11x - 4 = 0$

22. $x^2 + 8x - 84 = 0$

23. $x^2 - 7x + 5 = 0$

24. $x^2 + 3x - 8 = 0$

25. $x^2 - 5x - 10 = 0$

26. $3x^2 - 12x + 4 = 0$

27. $x^2 + 20x + 75 = 0$

28. $x^2 - 5x - 24 = 0$

29. $2x^2 + x - 21 = 0$

Lesson 6-5

(pages 313–319)

For Exercises 1–16, complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

1. $x^2 + 7x + 13 = 0$ **2.** $6x^2 + 6x - 21 = 0$ **3.** $5x^2 - 5x + 4 = 0$ **4.** $9x^2 + 42x + 49 = 0$

5. $4x^2 - 16x + 3 = 0$ **6.** $2x^2 = 5x + 3$ **7.** $x^2 + 81 = 18x$ **8.** $3x^2 - 30x + 75 = 0$

9. $24x^2 + 10x = 43$ **10.** $9x^2 + 4 = 2x$ **11.** $7x = 8x^2$ **12.** $18x^2 = 9x + 45$

13. $x^2 - 4x + 4 = 0$ **14.** $4x^2 + 16x + 15 = 0$ **15.** $x^2 - 6x + 13 = 0$ **16.** $3x^2 = 108x$

Solve each equation by using the method of your choice. Find the exact solutions.

17. $x^2 + 4x + 29 = 0$ **18.** $4x^2 + 3x - 2 = 0$ **19.** $2x^2 + 5x = 9$ **20.** $x^2 = 8x - 16$

21. $7x^2 = 4x$ **22.** $2x^2 + 6x + 5 = 0$ **23.** $9x^2 - 30x + 25 = 0$ **24.** $3x^2 - 4x + 2 = 0$

Extra Practice

Lesson 6-6

(pages 322–328)

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

1. $y = (x + 6)^2 - 1$ **2.** $y = 2(x - 8)^2 - 5$ **3.** $y = -(x + 1)^2 + 7$ **4.** $y = -9(x - 7)^2 + 3$

5. $y = -x^2 + 10x - 3$ **6.** $y = -2x^2 + 16x + 7$ **7.** $y = 3x^2 + 9x + 8$ **8.** $y = \frac{3}{4}x^2 - 6x - 5$

Graph each function.

9. $y = x^2 - 2x + 4$ **10.** $y = -3x^2 + 18x$ **11.** $y = -2x^2 - 4x + 1$ **12.** $y = 2x^2 - 8x + 9$

13. $y = \frac{1}{3}x^2 + 2x + 7$ **14.** $y = x^2 + 6x + 9$ **15.** $y = x^2 + 3x + 6$ **16.** $y = 2x^2 + 8x + 9$

17. $y = x^2 - 8x + 9$ **18.** $y = -x^2 - x + 10$ **19.** $y = -0.5x^2 + 4x - 3$ **20.** $y = -2x^2 - 8x - 1$

Write an equation for the parabola with the given vertex that passes through the given point.

21. vertex: $(-1, 5)$ point: $(2, -4)$ **22.** vertex: $(2, -1)$ point: $(-2, 7)$ **23.** vertex: $(-5, -3)$ point: $(-1, 5)$ **24.** vertex: $(0, -8)$ point: $(2, -2)$

Lesson 6-7

(pages 329–335)

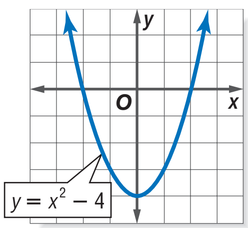
Graph each inequality.

1. $y \le 5x^2 + 3x - 2$ **2.** $y > -3x^2 + 2$ **3.** $y \ge x^2 - 8x$ **4.** $y \ge -x^2 - x + 3$

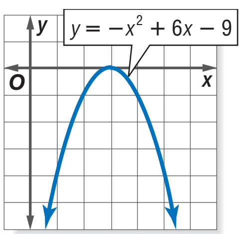
5. $y \le 3x^2 + 4x - 8$ **6.** $y \le -5x^2 + 2x - 3$ **7.** $y > 4x^2 + x$ **8.** $y \ge -x^2 - 3$

Use the graph of its related function to write the solutions of each inequality.

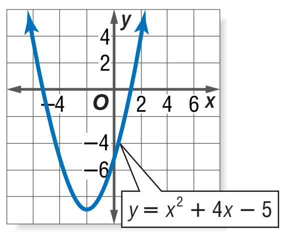
9. $x^2 - 4 \le 0$



10. $-x^2 + 6x - 9 \ge 0$



11. $x^2 + 4x - 5 < 0$



Solve each inequality algebraically.

12. $x^2 - 1 < 0$ **13.** $10x^2 - x - 2 \ge 0$ **14.** $-x^2 - 5x - 6 > 0$ **15.** $-3x^2 \ge 5$

16. $x^2 - 2x - 8 \le 0$ **17.** $2x^2 \ge 5x + 12$ **18.** $x^2 + 3x - 4 > 0$ **19.** $2x - x^2 \le -15$

Extra Practice

Lesson 7-1

(pages 346–352)

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. $2n^2 + 2m^2$
2. $5 - 3a^2$
3. $(x^2 + 2)(x^3 - 5)$
4. $-3a^3 + 5a - \frac{2}{a}$
5. $5c + c^2 - \frac{3}{5}$
6. $-4m - 7m^6 + 6m^5 + 3m^2$

Find $p(5)$ and $p(-1)$ for each function.

7. $p(x) = 7x - 3$
8. $p(x) = -3x^2 + 5x - 4$
9. $p(x) = 5x^4 + 2x^2 - 2x$
10. $p(x) = -13x^3 + 5x^2 - 3x + 2$
11. $p(x) = x^6 - 2$
12. $p(x) = \frac{2}{3}x^2 + 5x$
13. $p(x) = x^3 + x^2 - x + 1$
14. $p(x) = x^4 - x^2 - 1$
15. $p(x) = 1 - x^3$

If $p(x) = -2x^2 + 5x + 1$ and $q(x) = x^3 - 1$, find each value.

16. $q(n)$
17. $p(2b)$
18. $q(z^3)$
19. $p(3m^2)$
20. $q(x + 1)$
21. $p(3 - x)$
22. $q(a^2 - 2)$
23. $3q(h - 3)$
24. $5[p(c - 4)]$
25. $q(n - 2) + q(n^2)$
26. $-3p(4a) - p(a)$
27. $2[q(d^2 + 1)] + 3q(d)$

Lesson 7-2

(pages 353–358)

For Exercises 1–16, complete each of the following.
a. Graph each function by making a table of values.
b. Determine the values of x between which the real zeros are located.
c. Estimate the x-coordinates at which the relative maxima and relative minima occur.

1. $f(x) = x^3 + x^2 - 3x$
2. $f(x) = -x^4 + x^3 + 5$
3. $f(x) = x^3 - 3x^2 + 8x - 7$
4. $f(x) = 2x^5 + 3x^4 - 8x^2 + x + 4$
5. $f(x) = x^4 - 5x^3 + 6x^2 - x - 2$
6. $f(x) = 2x^6 + 5x^4 - 3x^2 - 5$
7. $f(x) = -x^3 - 8x^2 + 3x - 7$
8. $f(x) = -x^4 - 3x^3 + 5x$
9. $f(x) = x^5 - 7x^4 - 3x^3 + 2x^2 - 4x + 9$
10. $f(x) = x^4 - 5x^3 + x^2 - x - 3$
11. $f(x) = x^4 - 128x^2 + 960$
12. $f(x) = -x^5 + x^4 - 208x^2 + 145x + 9$
13. $f(x) = x^5 - x^3 - x + 1$
14. $f(x) = x^3 - 2x^2 - x + 5$
15. $f(x) = 2x^4 - x^3 + x^2 - x + 1$
16. $f(x) = -x^3 - x^2 - x - 1$

Lesson 7-3

(pages 360–364)

Write each polynomial in quadratic form, if possible.

1. $5x^{10} - 6x^5 - 3$
2. $2y^6 + 3y^4 + 10$
3. $z^6 - 8z^3$
4. $y^5 - 6y^3 + 4$
5. $x - 10x^{\frac{1}{2}} + 25$
6. $x^4 - 7x + 12$
7. $3r + 2r^{\frac{1}{2}} - 7$
8. $r^{\frac{2}{3}} - 5r^{\frac{1}{3}} + 6$
9. $x^{\frac{1}{2}} + 9x^{\frac{1}{4}} + 18$

Solve each equation.

10. $8x^3 - 27 = 0$
11. $2m^4 = 3m^2 + 5$
12. $3b^5 = 7b^3$
13. $x^3 + 10x^2 + 16x = 0$
14. $y^4 - 3y^2 + 2 = 0$
15. $a^3 = 125$
16. $5z^{\frac{2}{3}} - z^{\frac{1}{3}} = 4$
17. $x^{\frac{1}{2}} - 6x^{\frac{1}{4}} + 8 = 0$
18. $3m + m^{\frac{1}{2}} - 2 = 0$
19. $m - 9\sqrt{m} + 8 = 0$
20. $r^2 - 12r + 20 = 0$
21. $\sqrt[3]{x^2} - 8\sqrt[3]{x} + 15 = 0$
22. $m - 11\sqrt{m} + 30 = 0$
23. $y^3 - 8\sqrt{y^3} + 16 = 0$
24. $g^{\frac{2}{3}} - 2g^{\frac{1}{3}} - 8 = 0$

Lesson 7-4

(pages 365–370)

Use synthetic substitution to find $f(3)$ and $f(-4)$ for each function.

1. $f(x) = x^2 - 6x + 2$
2. $f(x) = x^3 + 5x - 6$
3. $f(x) = x^3 - x^2 - 3x + 1$
4. $f(x) = -3x^3 + 5x^2 + 7x - 3$
5. $f(x) = 3x^5 - 5x^3 + 2x - 8$
6. $f(x) = -2x^4 + 7x^3 + 8x^2 - 3x + 5$
7. $f(x) = 10x^3 + 2$
8. $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

9. $(x^3 - x^2 + x + 14); (x + 2)$
10. $(5x^3 - 17x^2 + 6x); (x - 3)$
11. $(2x^3 + x^2 - 41x + 20); (x - 4)$
12. $(x^3 - 8); (x - 2)$
13. $(x^2 + 6x + 5); (x + 1)$
14. $(x^4 + x^3 + x^2 + x); (x + 1)$
15. $(x^3 - 8x^2 + x + 42); (x - 7)$
16. $(6x^4 + 13x^3 - 36x^2 - 43x + 30); (x - 2)$
17. $(x^4 + 5x^3 - 27x - 135); (x - 3)$
18. $(2x^3 - 15x^2 - 2x + 120); (2x + 5)$
19. $(6x^3 - 17x^2 + 6x + 8); (3x - 4)$
20. $(30x^3 - 68x^2 + 10x + 12); (5x - 3)$
21. $(10x^3 + x^2 - 46x + 35); (5x - 7)$
22. $(x^3 + 9x^2 + 23x + 15); (x + 1)$

Lesson 7-5

(pages 371–377)

Solve each equation and state the number and type of roots.

1. $-5x - 7 = 0$
2. $3x^2 + 10 = 0$
3. $x^4 - 2x^3 - 7x^2 - 2x - 8 = 0$
4. $x^4 - 2x^3 = 23x^2 - 60x$

State the number of positive real zeros, negative real zeros, and imaginary zeros for each function.

5. $f(x) = 5x^8 - x^6 + 7x^4 - 8x^2 - 3$
6. $f(x) = 6x^5 - 7x^2 + 5$
7. $f(x) = -2x^6 - 5x^5 + 8x^2 - 3x + 1$
8. $f(x) = 4x^3 + x^2 - 38x + 56$
9. $f(x) = 3x^8 - 15x^5 - 7x^4 - 8x^3 - 3$
10. $f(x) = -x^6 - 8x^5 - 5x^4 - 11x^3 - 2x^2 - 5x - 1$
11. $f(x) = 3x^4 - 5x^3 + 2x^2 - 7x + 5$
12. $f(x) = x^5 - x^4 + 7x^3 - 25x^2 + 8x - 13$

Find all of the zeros of the function.

13. $f(x) = x^3 - 7x^2 + 16x - 10$
14. $f(x) = 10x^3 + 7x^2 - 82x + 56$
15. $f(x) = x^3 - 16x^2 + 79x - 114$
16. $f(x) = -3x^3 + 6x^2 + 5x - 8$
17. $f(x) = 6x^4 + 13x^3 - 18x^2 - 7x + 6$
18. $f(x) = 4x^4 + 36x^3 + 57x^2 + 225x + 200$
19. $f(x) = 24x^3 + 64x^2 + 6x - 10$
20. $f(x) = 2x^3 + 2x^2 - 34x + 30$

Write a polynomial function of least degree with integral coefficients that has the given zeros.

21. $-3, 1, 2$
22. $-5, -3, 3, 5$
23. $-6, 6, -5i, 5i$
24. $3, \pm i\sqrt{7}$

Lesson 7-6

(pages 378–382)

List all of the possible rational zeros for each function.

1. $f(x) = 3x^5 - 7x^3 - 8x + 6$
2. $f(x) = 4x^3 + 2x^2 - 5x + 8$
3. $f(x) = 6x^9 - 7$

Find all of the rational zeros for each function.

4. $f(x) = x^4 + 3x^3 - 7x^2 - 27x - 18$
5. $f(x) = 6x^4 - 31x^3 - 119x^2 + 214x + 560$
6. $f(x) = 20x^4 - 16x^3 + 11x^2 - 12x - 3$
7. $f(x) = 2x^4 - 30x^3 + 117x^2 - 75x + 280$
8. $f(x) = 3x^4 + 8x^3 + 9x^2 + 32x - 12$
9. $f(x) = 2x^6 - 12x^5 + 17x^4 + 6x^3 - 10x^2 + 6x - 9$
10. $f(x) = 2x^5 - 6x^4 + 18x^3 - 54x^2 - 20x + 60$
11. $f(x) = x^5 - x^4 + x^3 + 3x^2 - x$

Find all of the zeros of each function.

12. $f(x) = x^4 + 8x^2 - 9$
13. $f(x) = 2x^4 - 6x^3 - 70x^2 - 30x - 400$
14. $f(x) = 3x^4 - 9x^2 - 12$
15. $f(x) = 4x^4 + 19x^2 - 63$

Extra Practice

Lesson 7-7
(pages 383–389)

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$.

1. $f(x) = 3x + 5$, $g(x) = x - 3$
2. $f(x) = \sqrt{x}$, $g(x) = x^2$
3. $f(x) = 2x^2 - 5x + 8$, $g(x) = \frac{x - 8}{3}$
4. $f(x) = x^2 - 5$, $g(x) = x^2 + 5$
5. $f(x) = x + 2$, $g(x) = x^2 + 4x + 4$
6. $f(x) = x^2 + 1$, $g(x) = x + 1$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

7. $f = \{(-1, 1), (2, -1), (-3, 5)\}$, $g = \{(1, -1), (-1, 2), (5, -3)\}$
8. $f = \{(0, 6), (5, -8), (-9, 2)\}$, $g = \{(-8, 3), (6, 4), (2, 1)\}$
9. $f = \{(8, 2), (6, 5), (-3, 4), (1, 0)\}$, $g = \{(2, 8), (5, 6), (4, -3), (0, 1)\}$
10. $f = \{(10, 4), (-1, 2), (5, 6), (-1, 0)\}$, $g = \{(-4, 10), (2, -9), (-7, 5), (-2, -1)\}$

Find $[g \circ h](x)$ and $[h \circ g](x)$.

11. $g(x) = 8 - 2x$, $h(x) = 3x$
12. $g(x) = x^2 - 7$, $h(x) = 3x + 2$
13. $g(x) = 2x + 7$, $h(x) = \frac{x - 7}{2}$
14. $g(x) = 3x + 2$, $h(x) = 5 - 3x$

If $f(x) = x^2 + 1$, $g(x) = 2x$, and $h(x) = x - 1$, find each value.

15. $g[f(1)]$
16. $[f \circ h](3)$
17. $[h \circ f](3)$
18. $[g \circ f](-2)$
19. $g[h(-20)]$
20. $f[h(-3)]$
21. $g[f(a)]$
22. $[f \circ (g \circ f)](c)$

Lesson 7-8
(pages 390–394)

Find the inverse of each relation.

1. $\{(-2, 7), (3, 0), (5, -8)\}$
2. $\{(-3, 9), (-2, 4), (3, 9), (-1, 1)\}$
3. $\{(1, 5), (2, 3), (4, 3), (-1, 5)\}$

Find the inverse of each function. Then graph the function and its inverse.

4. $f(x) = x - 7$
5. $y = 2x + 8$
6. $g(x) = 3x - 8$
7. $h(x) = \frac{x}{5} + 1$
8. $y = -2$
9. $g(x) = 5 - 2x$
10. $y = -5x - 6$
11. $h(x) = -\frac{2}{3}x$
12. $y = \frac{x - 5}{3}$
13. $y = \frac{1}{2}x - 1$
14. $f(x) = \frac{3x + 8}{4}$
15. $g(x) = \frac{2x - 1}{3}$

Determine whether each pair of functions are inverse functions.

16. $f(x) = \frac{2x - 3}{5}$, $g(x) = \frac{3x - 5}{3}$
17. $f(x) = 5x - 6$, $g(x) = \frac{x + 6}{5}$
18. $f(x) = 6 - 3x$, $g(x) = 2 - \frac{1}{3}x$
19. $f(x) = 3x - 7$, $g(x) = \frac{1}{3}x + 7$

Lesson 7-9
(pages 395–399)

Graph each function. State the domain and range of each function.

1. $y = \sqrt{x - 4}$
2. $y = \sqrt{x + 3} - 1$
3. $y = \frac{1}{3}\sqrt{x + 2}$
4. $y = \sqrt{2x + 5}$
5. $y = -\sqrt{4x}$
6. $y = 2\sqrt{x}$
7. $y = -3\sqrt{x}$
8. $y = \sqrt{x} + 5$
9. $y = \sqrt{2x} - 1$
10. $y = 5\sqrt{x} + 1$
11. $y = \sqrt{x + 1} - 2$
12. $y = 6 - \sqrt{x + 3}$

Graph each inequality.

13. $y > \sqrt{2x}$
14. $y \leq \sqrt{-5x}$
15. $y \geq \sqrt{x + 6} + 6$
16. $y < \sqrt{3x + 1} + 2$
17. $y \geq \sqrt{8x - 3} + 1$
18. $y < \sqrt{5x - 1} + 3$

Lesson 8-1

(pages 412–416)

Find the midpoint of the line segment with endpoints at the given coordinates.

1. $(7, -3), (-11, 13)$
2. $(16, 29), (-7, 2)$
3. $(43, -18), (-78, -32)$
4. $(-7.54, 3.42), (4.89, -9.28)$
5. $\left(\frac{1}{2}, \frac{1}{4}\right), \left(\frac{2}{3}, \frac{3}{5}\right)$
6. $\left(-\frac{1}{4}, \frac{2}{3}\right), \left(-\frac{1}{2}, -\frac{1}{2}\right)$

Find the distance between each pair of points with the given coordinates.

7. $(5, 7), (3, 19)$
8. $(-2, -1), (5, 3)$
9. $(-3, 15), (7, -8)$
10. $(6, -3), (-4, -9)$
11. $(3.89, -0.38), (4.04, -0.18)$
12. $(5\sqrt{3}, 2\sqrt{2}), (-11\sqrt{3}, -4\sqrt{2})$
13. $\left(\frac{1}{4}, 0\right), \left(-\frac{2}{3}, \frac{1}{2}\right)$
14. $\left(4, -\frac{5}{6}\right), \left(-2, \frac{1}{6}\right)$
15. A circle has a radius with endpoints at $(-3, 1)$ and $(2, -5)$. Find the circumference and area of the circle. Write the answer in terms of π.
16. Triangle *ABC* has vertices $A(0, 0)$, $B(-3, 4)$, and $C(2, 6)$. Find the perimeter of the triangle.

Lesson 8-2

(pages 419–425)

Write each equation in standard form.

1. $y = x^2 - 4x + 7$
2. $y = 2x^2 + 12x + 17$
3. $x = 3y^2 - 6y + 5$

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.

4. $y + 4 = x^2$
5. $y = 5(x + 2)^2$
6. $4(y + 2) = 3(x - 1)^2$
7. $5x + 3y^2 = 15$
8. $y = 2x^2 - 8x + 7$
9. $x = 2y^2 - 8y + 7$
10. $3(x - 8)^2 = 5(y + 3)$
11. $x = 3(y + 4)^2 + 1$
12. $8y + 5x^2 + 30x + 101 = 0$
13. $x = -\frac{1}{5}y^2 + \frac{8}{5}y - 7$
14. $6x = y^2 - 6y + 39$
15. $-8y = x^2$
16. $y = 4x^2 + 24x + 38$
17. $y = x^2 - 6x + 3$
18. $y = x^2 + 4x + 1$

Write an equation for each parabola described below. Then draw the graph.

19. focus $(1, 1)$, directrix $y = -1$
20. vertex $(-1, 2)$, directrix $y = -4$
21. vertex $(2, -3)$, focus $(0, -3)$

Lesson 8-3

(pages 426–431)

Write an equation for the circle that satisfies each set of conditions.

1. center $(3, 2)$, $r = 5$ units
2. center $(-5, 8)$, $r = 3$ units
3. center $(1, -6)$, $r = \frac{2}{3}$ units
4. center $(0, 7)$, tangent to x-axis
5. center $(-2, -4)$, tangent to y-axis
6. endpoints of a diameter at $(-9, 0)$ and $(2, -5)$
7. endpoints of a diameter at $(4, 1)$ and $(-3, 2)$
8. center $(6, -10)$, passes through origin
9. center $(0.8, 0.5)$, passes through $(2, 2)$

Find the center and radius of the circle with the given equation. Then graph the circle.

10. $x^2 + y^2 = 36$
11. $\left(x - \frac{3}{4}\right)^2 + \left(y + \frac{2}{3}\right)^2 = \frac{32}{49}$
12. $(x - 5)^2 + (y + 4)^2 = 1$
13. $x^2 + 3x + y^2 - 5y = 0.5$
14. $x^2 + y^2 = 14x - 24$
15. $x^2 + y^2 = 2(y - x)$
16. $x^2 + 10x + (y - \sqrt{3})^2 = 11$
17. $x^2 + y^2 = 4x + 9$
18. $x^2 + y^2 + 12x - 10y + 45 = 0$
19. $x^2 + y^2 - 6x + 4y = 156$
20. $x^2 + y^2 - 2x + 7y = 1$
21. $16(x^2 + y^2) - 8(3x + 5y) + 33 = 0$

Extra Practice

Extra Practice

Lesson 8-4

(pages 433–440)

Write an equation for the ellipse that satisfies each set of conditions.

1. endpoints of major axis at $(-2, 7)$ and $(4, 7)$, endpoints of minor axis at $(1, 5)$ and $(1, 9)$
2. endpoints of minor axis at $(1, -4)$ and $(1, 5)$, endpoints of major axis at $(-4, 0.5)$ and $(6, 0.5)$
3. major axis 24 units long and parallel to the y-axis, minor axis 4 units long, center at $(0, 3)$

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

4. $\frac{x^2}{36} + \frac{y^2}{81} = 1$
5. $\frac{x^2}{121} + \frac{(y-5)^2}{16} = 1$
6. $\frac{(x+2)^2}{12} + \frac{(y+1)^2}{16} = 1$
7. $\frac{(x+2)^2}{36} + \frac{(y-4)^2}{40} = 1$
8. $\frac{(x+8)^2}{121} + \frac{(y-7)^2}{64} = 1$
9. $\frac{(x-4)^2}{16} + \frac{(y+1)^2}{9} = 1$
10. $8x^2 + 2y^2 = 32$
11. $7x^2 + 3y^2 = 84$
12. $9x^2 + 16y^2 = 144$
13. $169x^2 - 338x + 169 + 25y^2 = 4225$
14. $x^2 + 4y^2 + 8x - 64y = -128$
15. $4x^2 + 5y^2 = 6(6x + 5y) + 658$
16. $9x^2 + 16y^2 - 54x + 64y + 1 = 0$

Lesson 8-5

(pages 441–448)

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

1. $\frac{y^2}{25} - \frac{x^2}{9} = 1$
2. $\frac{x^2}{4} - \frac{y^2}{9} = 1$
3. $\frac{x^2}{81} - \frac{y^2}{36} = 1$
4. $\frac{x^2}{9} - \frac{y^2}{16} = 1$
5. $\frac{y^2}{100} - \frac{x^2}{144} = 1$
6. $\frac{x^2}{16} - \frac{y^2}{4} = 1$
7. $\frac{(x-4)^2}{64} - \frac{(y+1)^2}{16} = 1$
8. $\frac{(y-7)^2}{2.25} - \frac{(x-3)^2}{4} = 1$
9. $(x+5)^2 - \frac{(y+3)^2}{48} = 1$
10. $x^2 - 9y^2 = 36$
11. $4x^2 - 9y^2 = 72$
12. $49x^2 - 16y^2 = 784$
13. $144x^2 + 1152x - 25y^2 - 100y = 1396$
14. $576y^2 = 49x^2 + 490x + 29449$
15. $23.04y^2 - 46.08y - 1.96x^2 - 3.92x = 24.0784$
16. $25(y+5)^2 - 20(x-1)^2 = 500$

Write an equation for the hyperbola that satisfies each set of conditions.

17. vertices $(-3, 0)$ and $(3, 0)$; conjugate axis of length 8 units
18. vertices $(0, -7)$ and $(0, 7)$; conjugate axis of length 25 units
19. center $(0, 0)$; horizontal transverse axis of length 12 units and a conjugate axis of length 10 units

Lesson 8-6

(pages 449–452)

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

1. $9x^2 - 36x + 36 = 4y^2 + 24y + 72$
2. $x^2 + 4x + 2y^2 + 16y + 32 = 0$
3. $x^2 + 6x + y^2 - 6y + 9 = 0$
4. $9y^2 = 25x^2 + 400x + 1825$
5. $2y^2 + 12y - x + 6 = 0$
6. $x^2 + y^2 = 10x + 2y + 23$
7. $3x^2 + y = 12x - 17$
8. $9x^2 - 18x + 16y^2 + 160y = -265$
9. $x^2 + 10x + 5 = 4y^2 + 16$
10. $\frac{(y-5)^2}{4} - (x+1)^2 = 4$
11. $9x^2 + 49y^2 = 441$
12. $4x^2 - y^2 = 4$

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

13. $(x+3)^2 = 8(y+2)$
14. $x^2 + 4x + y^2 - 8y = 2$
15. $9x^2 + 9y^2 = 9$
16. $y - x^2 = x + 3$
17. $2x^2 - 13y^2 + 5 = 0$
18. $16(x-3)^2 + 81(y+4)^2 = 1296$
19. $x^2 + 5y^2 = 16$
20. $4x^2 - y^2 = 16$

Lesson 8-7

(pages 455–460)

Solve each system of inequalities by graphing.

1. $\frac{x^2}{16} - \frac{y^2}{1} \geq 1$, $x^2 + y^2 \leq 49$
2. $\frac{x^2}{25} + \frac{y^2}{16} \leq 1$, $y \leq x - 2$
3. $y \geq x + 3$, $x^2 + y^2 < 25$
4. $4x^2 + (y - 3)^2 \leq 16$, $x + 2y \geq 4$

Find the exact solution(s) of each system of equations.

5. $\frac{x^2}{16} + \frac{y^2}{16} = 1$, $y = x + 3$
6. $x = y^2$, $(x + 3)^2 + y^2 = 53$
7. $\frac{x^2}{3} - \frac{(y + 2)^2}{4} = 1$, $x^2 = y^2 + 11$
8. $\frac{(x - 1)^2}{5} + \frac{y^2}{2} = 1$, $y = x + 1$
9. $x^2 + y^2 = 13$, $x^2 - y^2 = -5$
10. $\frac{x^2}{25} - \frac{y^2}{5} = 1$, $y = x - 4$
11. $x^2 + y = 0$, $x + y = -2$
12. $x^2 - 9y^2 = 36$, $x = y$
13. $4x^2 + 6y^2 = 360$, $y = x$

Lesson 9-1

(pages 472–478)

Simplify each expression.

1. $\frac{25xy^2}{15y}$
2. $\frac{-4a^2b^3}{28ab^4}$
3. $\frac{(-2cd^3)^2}{8c^2d^5}$
4. $\frac{3x^3}{-2} \cdot \frac{-4}{9x}$
5. $\frac{21x^2}{-5} \cdot \frac{10}{7x^3}$
6. $\frac{2u^2}{3} \div \frac{6u^3}{5}$
7. $\frac{15x^3}{14} \div \frac{18x}{7}$
8. $\frac{xy^2}{2} \cdot \frac{x^2}{2y} \cdot \frac{2}{x^2y}$
9. $axy \div \frac{ax}{y}$
10. $\frac{9u^2}{28v} \div \frac{27u^2}{8v^2}$
11. $\frac{x^2 - 4}{4x^2 - 1} \cdot \frac{2x - 1}{x + 2}$
12. $\frac{x^2 - 1}{2x^2 - x - 1} \div \frac{x^2 - 4}{2x^2 - 3x - 2}$
13. $\frac{2x^2 + x - 1}{2x^2 + 3x - 2} \div \frac{x^2 - 2x + 1}{x^2 + x - 2}$
14. $\frac{\frac{(ab)^2}{c}}{\frac{xa^3b}{cx^2}}$
15. $\frac{\frac{x^4 - y^4}{x^3 + y^3}}{\frac{x^3 - y^3}{x + y}}$

Lesson 9-2

(pages 479–484)

Find the LCM of each set of polynomials.

1. $2a^2b, 4ab^2, 20a$
2. $48c^2d, 72cd^2$
3. $x^2 - 4x - 12, x^2 + 7x + 10$

Simplify each expression.

4. $\frac{12}{7d} - \frac{3}{14d}$
5. $\frac{x + 1}{x} - \frac{x - 1}{x^2}$
6. $\frac{2x + 1}{4x^2} - \frac{x + 3}{6x}$
7. $\frac{7x}{13y^2} + \frac{4y}{6x^2}$
8. $\frac{x}{x - 1} + \frac{1}{1 - x}$
9. $\frac{1}{3v^2} + \frac{1}{uv} + \frac{3}{4u^2}$
10. $\frac{1}{x^2 - x} + \frac{1}{x^2 + x}$
11. $\frac{1}{x^2 - 1} - \frac{1}{(x - 1)^2}$
12. $\frac{5}{x} - \frac{3}{x + 5}$
13. $y - 1 + \frac{1}{y - 1}$
14. $3m + 1 - \frac{2m}{3m + 1}$
15. $\frac{3x}{x - y} + \frac{4x}{y - x}$
16. $\frac{6}{4m^2 - 12mn + 9n^2} + \frac{2}{2mn - 3n^2}$
17. $\frac{3}{x^2 + 5ax + 6a^2} + \frac{2}{x^2 - 4a^2}$
18. $\frac{x}{x^2 + 5x + 6} - \frac{2}{x^2 + 4x + 4}$
19. $\frac{4}{a^2 - 4} - \frac{3}{a^2 + 4a + 4}$
20. $\frac{4}{3 - 3z^2} - \frac{2}{z^2 + 5z + 4}$
21. $\frac{2c}{c^2 - 9} - \frac{1}{c^2 + 6c + 9}$
22. $\frac{\frac{1}{x + y}}{\frac{1}{x} + \frac{1}{y}}$
23. $\frac{1 - \frac{1}{x + 1}}{1 + \frac{1}{x - 1}}$
24. $\frac{4 + \frac{1}{x - 2}}{3 - \frac{1}{x - 2}}$

Extra Practice

Lesson 9-3

(pages 485–490)

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{1}{x + 4}$
2. $f(x) = \frac{x - 2}{x + 3}$
3. $f(x) = \frac{5}{(x + 1)(x - 8)}$
4. $f(x) = \frac{x}{x + 2}$
5. $f(x) = \frac{x^2 - 4}{x + 2}$
6. $f(x) = \frac{x^2 + x - 6}{x^2 + 8x + 15}$

Graph each rational function.

7. $f(x) = \frac{1}{x - 5}$
8. $f(x) = \frac{3x}{x + 1}$
9. $f(x) = \frac{x^2 - 16}{x - 4}$
10. $f(x) = \frac{x}{x - 6}$
11. $f(x) = \frac{1}{(x - 3)^2}$
12. $f(x) = \frac{2}{(x + 3)(x - 4)}$
13. $f(x) = \frac{x + 4}{x^2 - 1}$
14. $f(x) = \frac{x + 2}{x + 3}$
15. $f(x) = \frac{x^2 + 5x - 14}{x^2 + 9x + 14}$

Lesson 9-4

(pages 492–498)

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

1. $xy = 10$
2. $x = 6y$
3. $\frac{x}{7} = y$
4. $\frac{x}{y} = -6$
5. $10x = y$
6. $x = \frac{2}{y}$
7. $A = lw$
8. $\frac{1}{4}b = -\frac{3}{5}c$
9. $D = rt$

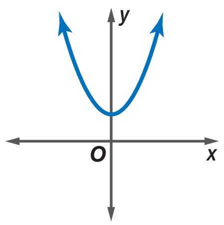
Find each value.

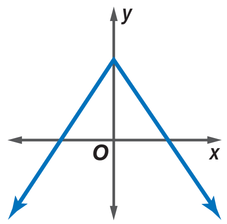
10. If y varies directly as x and $y = 16$ when $x = 4$, find y when $x = 12$.
11. If x varies inversely as y and $x = 12$ when $y = -3$, find x when $y = -18$.
12. If m varies directly as w and $m = -15$ when $w = 2.5$, find m when $w = 12.5$.
13. If y varies jointly as x and z and $y = 10$ when $z = 4$ and $x = 5$, find y when $x = 4$ and $z = 2$.
14. If y varies inversely as x and $y = \frac{1}{4}$ when $x = 24$, find y when $x = \frac{3}{4}$.
15. If y varies jointly as x and z and $y = 45$ when $x = 9$ and $z = 15$, find y when $x = 25$ and $z = 12$.

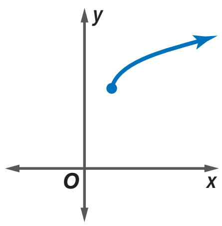
Lesson 9-5

(pages 499–504)

Identify the type of function represented by each graph.

1.


2.


3.


Identify the function represented by each equation. Then graph the equation.

4. $y = \sqrt{5x}$
5. $y = \frac{3}{4}x$
6. $y = |x| + 3$
7. $y = x^2 - 2$
8. $y = \frac{2}{x}$
9. $y = 2[\![x]\!]$
10. $y = -2x^2 + 1$
11. $y = \frac{x^2 + 2x - 3}{x^2 + 7x + 12}$
12. $y = -3$

CONTENTS

Lesson 9-6

(pages 505–511)

Solve each equation or inequality. Check your solutions.

1. $\frac{x}{x-3} = \frac{1}{4}$
2. $\frac{5}{x} + \frac{3}{5} = \frac{2}{x}$
3. $\frac{5}{b-2} < 5$
4. $\frac{4}{a+3} > 2$
5. $\frac{x-2}{x} = \frac{x-4}{x-6}$
6. $-6 - \frac{8}{n} < n$
7. $\frac{2}{d} + \frac{1}{d-2} = 1$
8. $\frac{1}{2+3x} + \frac{2}{2-3x} = 0$
9. $\frac{1}{n+1} + \frac{1}{n-1} = \frac{2}{n^2-1}$
10. $\frac{1}{x-3} + \frac{1}{x+5} = \frac{x+1}{x-3}$
11. $\frac{4}{x^2-2x-3} = \frac{-x}{3-x} - \frac{1}{x+1}$
12. $\frac{p}{p+1} + \frac{3}{p-3} + 1 = 0$
13. $\frac{3x}{x^2+2x-8} = \frac{1}{x-2} + \frac{x}{x+4}$
14. $\frac{5z+2}{z^2-4} = \frac{-5z}{2-z} + \frac{2}{z+2}$
15. $\frac{1}{x-3} + \frac{2}{x^2-9} = \frac{5}{x+3}$
16. $\frac{1}{m^2-1} = \frac{2}{m^2+m-2}$
17. $\frac{12}{x^2-16} - \frac{24}{x-4} = 3$
18. $n + \frac{1}{n+3} = \frac{n^2}{n-1}$

Lesson 10-1

(pages 523–530)

Sketch the graph of each function. Then state the function's domain and range.

1. $y = 3(5)^x$
2. $y = 0.5(2)^x$
3. $y = 3\left(\frac{1}{4}\right)^x$
4. $y = 2(1.5)^x$

Determine whether each function represents exponential growth or decay.

5. $y = 4(3)^x$
6. $y = 10^{-x}$
7. $y = 5\left(\frac{1}{2}\right)^x$
8. $y = 2\left(\frac{5}{4}\right)^x$

Write an exponential function whose graph passes through the given points.

9. (0, 6) and (2, 54)
10. (0, −4) and (−4, −64)
11. (0, 1.5) and (3, 40.5)
12. (0, −3.7) and (5, −118.4)

Simplify each expression.

13. $4^{\sqrt{2}} \cdot 4^{\sqrt{8}}$
14. $\left(5^{\sqrt{5}}\right)^{\sqrt{45}}$
15. $\left(w^{\sqrt{6}}\right)^{\sqrt{3}}$
16. $27^{\sqrt{5}} \div 3^{\sqrt{5}}$
17. $8^{2\sqrt{3}} \cdot 4^{\sqrt{3}}$
18. $5^{\sqrt{2}} \cdot 5^{\sqrt{3}}$
19. $7^{\sqrt{3}} \cdot 7^{2\sqrt{3}}$
20. $\left(y^{\sqrt{3}}\right)^{\sqrt{27}}$

Solve each equation or inequality. Check your solution.

21. $27^{2x-1} = 3$
22. $8^{2+x} \geq 2$
23. $4^{2x+5} < 8^{x+1}$
24. $6^{x+1} = 36^{x-1}$
25. $10^{x-1} > 100^{4-x}$
26. $\left(\frac{1}{5}\right)^{x-3} = 125$
27. $2^{x^2+1} = 32$
28. $36^x = 6^{x^2-3}$

Lesson 10-2

(pages 531–538)

Write each equation in logarithmic form.

1. $3^5 = 243$
2. $10^3 = 1000$
3. $4^{-3} = \frac{1}{64}$

Write each equation in exponential form.

4. $\log_2 \frac{1}{8} = -3$
5. $\log_{25} 5 = \frac{1}{2}$
6. $\log_7 \frac{1}{7} = -1$

Evaluate each expression.

7. $\log_4 16$
8. $\log_{10} 10{,}000$
9. $\log_3 \frac{1}{9}$
10. $\log_2 1024$
11. $\log_6 6^5$
12. $\log_{\frac{1}{2}} 8$
13. $\log_{11} 121$
14. $5^{\log_5 10}$

Solve each equation or inequality. Check your solution.

15. $\log_8 b = 2$
16. $\log_4 x < 3$
17. $\log_{\frac{1}{9}} n = -\frac{1}{2}$
18. $\log_x 7 = 1$
19. $\log_{\frac{2}{3}} a < 3$
20. $\log_2 (x^2 - 9) = 4$

Extra Practice

Lesson 10-3

(pages 541–546)

Use $\log_3 5 \approx 1.4651$ and $\log_3 7 \approx 1.7712$ to approximate the value of each expression.

1. $\log_3 \frac{7}{5}$
2. $\log_3 245$
3. $\log_3 35$

Solve each equation. Check your solutions.

4. $\log_2 x + \log_2 (x - 2) = \log_2 3$
5. $\log_3 x = 2 \log_3 3 + \log_3 5$
6. $\log_5 (x^2 + 7) = \frac{2}{3} \log_5 64$
7. $\log_7 (3x + 5) - \log_7 (x - 5) = \log_7 8$
8. $\log_2 (x^2 - 9) = 4$
9. $\log_3 (x + 2) + \log_3 6 = 3$
10. $\log_6 x + \log_6 (x - 5) = 2$
11. $\log_5 (x + 3) = \log_5 8 - \log_5 2$
12. $2 \log_3 x - \log_3 (x - 2) = 2$
13. $\log_6 x = \frac{3}{2} \log_6 9 + \log_6 2$
14. $\log_8 (x + 6) + \log_8 (x - 6) = 2$
15. $\frac{1}{2}\log_4 (x + 2) + \frac{1}{2}\log_4 (x + 2) = \frac{2}{3}\log_4 27$
16. $\log_3 14 + \log_3 x = \log_3 42$
17. $\log_{10} x = \frac{1}{2}\log_{10} 81$

Lesson 10-4

(pages 547–551)

Use a calculator to evaluate each expression to four decimal places.

1. $\log 55$
2. $\log 6.7$
3. $\log 3.3$
4. $\log 0.08$
5. $\log 9.9$
6. $\log 0.6$

Solve each equation or inequality. Round to four decimal places.

7. $2^x = 15$
8. $4^{2a} > 45$
9. $7^{2x} = 35$
10. $11^{x + 4} > 57$
11. $1.5^{a - 7} = 9.6$
12. $3^{b^2} = 64$
13. $7^{3c} < 35^{2c - 1}$
14. $5^{m^2 + 1} = 30$
15. $7^{3y - 1} < 2^{2y + 4}$
16. $9^{n - 3} = 2^{n + 3}$
17. $11^{t + 1} \leq 22^{t + 3}$
18. $2^{3a - 1} = 3^{a + 2}$

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

19. $\log_3 21$
20. $\log_4 62$
21. $\log_5 28$
22. $\log_2 25$
23. $\log_{12} 30$
24. $\log_4 63$
25. $\log_7 35$
26. $\log_6 100$

Lesson 10-5

(pages 554–559)

Use a calculator to evaluate each expression to four decimal places.

1. e^3
2. $e^{0.75}$
3. e^{-4}
4. $e^{-2.5}$
5. $\ln 5$
6. $\ln 8$
7. $\ln 8.4$
8. $\ln 0.6$

Write an equivalent exponential or logarithmic equation.

9. $e^x = 10$
10. $\ln x \approx 2.3026$
11. $e^3 = 9x$
12. $\ln 0.2 = x$

Evaluate each expression.

13. $e^{\ln 5}$
14. $\ln e^{x - 2}$
15. $\ln e^{-3n}$
16. $e^{\ln 2.5}$

Solve each equation or inequality.

17. $25e^x = 1000$
18. $e^{0.075x} > 25$
19. $e^x < 3.8$
20. $-2e^x + 5 = 1$
21. $5 + 4e^{2x} = 17$
22. $e^{-3x} \leq 15$
23. $\ln 7x = 10$
24. $\ln 4x = 8$
25. $3 \ln 2x \geq 9$
26. $\ln (x + 2) = 4$
27. $\ln (2x + 3) > 0$
28. $\ln (3x - 1) = 5$

Lesson 10-6

(pages 560–565)

1. Mr. Rogers purchased a combine for \$175,000 for his farming operation. It is expected to depreciate at a rate of 18% per year. What will be the value of the combine in 3 years?
2. The Jacksons bought a house for \$65,000 in 1992. Houses in the neighborhood have appreciated at the rate of 4.5% a year. How much is the house worth in 2003?
3. In 1950, the population of a city was 50,000. Since then, the population has increased by 2.25% per year. If it continues to grow at this rate, what will the population be in 2005?
4. In a particular state, the population of black bears has been decreasing at the rate of 0.75% per year. In 1990, it was estimated that there were 400 black bears in the state. If the population continues to decline at the same rate, what will the population be in 2010?
5. Scott invests \$8500 in a certificate of deposit (CD). The interest is calculated on the money once per year. If the interest rate for the CD is 6.875% and he invests in a 4-year CD, how much money will he have at the end of the investment period?
6. Mrs. Nguyen takes a job as a teacher with a starting salary of \$20,050. Over the past few years in this district, teachers have received a 4% increase in pay each year. If the pay continues to increase at the same rate, what will be her salary in 5 years?

Lesson 11-1

(pages 578–582)

Find the next four terms of each arithmetic sequence.

1. 9, 7, 5, …
2. 3, 4.5, 6, …
3. 40, 35, 30, …
4. 2, 5, 8, …

Find the first five terms of each arithmetic sequence described.

5. $a_1 = 1, d = 7$
6. $a_1 = -5, d = 2$
7. $a_1 = 1.2, d = 3.7$
8. $a_1 = -\frac{5}{4}, d = -\frac{1}{2}$

Find the indicated term of each arithmetic sequence.

9. $a_1 = 4, d = 5, n = 10$
10. $a_1 = -30, d = -6, n = 5$
11. $a_1 = -3, d = 32, n = 8$
12. $a_1 = \frac{3}{4}, d = -\frac{1}{4}, n = 72$
13. $a_1 = -\frac{1}{5}, d = \frac{3}{5}, n = 17$
14. $a_1 = 20, d = -3, n = 16$

Write an equation for the *n*th term of each arithmetic sequence.

15. 3, 5, 7, 9, …
16. 2, −1, −4, −7, …
17. 20, 28, 36, 44, …

Find the arithmetic means in each sequence.

18. 2, _?_, _?_, _?_, 34
19. 0, _?_, _?_, _?_, −28
20. −10, _?_, _?_, _?_, 14

Lesson 11-2

(pages 583–587)

Find S_n for each arithmetic series described.

1. $a_1 = 3, a_n = 20, n = 6$
2. $a_1 = 15, a_n = -12, n = 30$
3. $a_1 = 90, a_n = -4, n = 10$
4. $a_1 = 16, a_n = 14, n = 12$
5. $a_1 = -80, a_n = 120, n = 18$
6. $a_1 = -3, a_n = -72, n = 14$
7. $a_1 = -1, d = 10, n = 30$
8. $a_1 = 4, d = -5, n = 11$
9. $a_1 = 5, d = -\frac{1}{2}, n = 17$

Find the sum of each arithmetic series.

10. $3 + 12 + 21 + 30 + \ldots + 57$
11. $1 + 4 + 7 + 10 + \ldots + 31$
12. $8 + 16 + 24 + \ldots + 80$
13. $\sum_{n=1}^{6} (n + 2)$
14. $\sum_{n=5}^{10} (2n - 5)$
15. $\sum_{k=1}^{5} (40 - 2k)$
16. $\sum_{k=8}^{12} (6 - 3k)$
17. $\sum_{n=1}^{4} (10n + 2)$
18. $\sum_{n=6}^{10} (2 + 3n)$

Find the first three terms of each arithmetic series described.

19. $a_1 = 11, a_n = 38, S_n = 245$
20. $a_1 = -6, a_n = -34, S_n = -300$
21. $a_1 = 0, a_n = 165, S_n = 990$
22. $n = 12, a_n = 13, S_n = -42$
23. $n = 11, a_n = 5, S_n = 0$
24. $a_1 = -25, a_n = 15, S_n = -45$

Extra Practice

Lesson 11-3

(pages 588–592)

Find the next two terms of each geometric sequence.

1. 5, 15, 45, …
2. 2, 10, 50, …
3. 64, 16, 4, …
4. −9, 27, −81, …
5. 0.5, 0.75, 1.125, …
6. $\frac{1}{2}, -\frac{3}{8}, \frac{9}{32}, \ldots$

Find the first five terms of each geometric sequence described.

7. $a_1 = -2, r = 6$
8. $a_1 = 4, r = -5$
9. $a_1 = 1, r = -3$
10. $a_1 = 1, r = 0.8$
11. $a_1 = 0.8, r = 2.5$
12. $a_1 = 1.5, r = -3$
13. $a_1 = \frac{2}{5}, r = -3$
14. $a_1 = -\frac{1}{3}, r = -\frac{3}{5}$

Find the indicated term of each geometric sequence.

15. $a_1 = 5, r = 7, n = 6$
16. $a_1 = 200, r = -\frac{1}{2}, n = 10$
17. $a_1 = 60, r = -2, n = 4$
18. $a_1 = 300, r = \frac{1}{4}, n = 6$
19. $a_1 = 8, r = -2, n = 8$
20. $a_1 = 1, r = -1, n = 30$

Write an equation for the *n*th term of each geometric sequence.

21. 20, 40, 80, …
22. $-\frac{1}{2}, -\frac{1}{8}, -\frac{1}{32}, \ldots$

Find the geometric means in each sequence.

23. 1, __?__, __?__, __?__, 81
24. 5, __?__, __?__, __?__, 6480

Lesson 11-4

(pages 594–598)

Find S_n for each geometric series described.

1. $a_1 = \frac{1}{81}, r = 3, n = 6$
2. $a_1 = 1, r = -2, n = 7$
3. $a_1 = 5, r = 4, n = 5$
4. $a_1 = -27, r = -\frac{1}{3}, n = 6$
5. $a_1 = 1000, r = \frac{1}{2}, n = 7$
6. $a_1 = 125, r = -\frac{2}{5}, n = 5$
7. $a_1 = 10, r = 3, n = 6$
8. $a_1 = 1250, r = -\frac{1}{5}, n = 5$
9. $a_1 = 1215, r = \frac{1}{3}, n = 5$
10. $a_1 = 16, r = \frac{3}{2}, n = 5$
11. $a_1 = 7, r = 2, n = 7$
12. $a_1 = -\frac{3}{2}, r = -\frac{1}{2}, n = 6$

Find the sum of each geometric series.

13. $\sum_{k=1}^{5} 2^k$
14. $\sum_{n=0}^{3} 3^{-n}$
15. $\sum_{n=0}^{3} 2(5^n)$
16. $\sum_{k=2}^{5} -(-3)^{k-1}$

Find the indicated term for each geometric series described.

17. $S_n = 300, a_n = 160, r = 2; a_1$
18. $S_n = -171, n = 9, r = -2; a_5$
19. $S_n = -4372, a_n = -2916, r = 3; a_4$

Lesson 11-5

(pages 599–604)

Find the sum of each infinite geometric series, if it exists.

1. $a_1 = 54, r = \frac{1}{3}$
2. $a_1 = 2, r = -1$
3. $a_1 = 1000, r = -0.2$
4. $a_1 = 7, r = \frac{3}{7}$
5. $49 + 14 + 4 + \ldots$
6. $\frac{3}{4} + \frac{1}{2} + \frac{1}{3} + \ldots$
7. $12 - 4 + \frac{4}{3} - \ldots$
8. $3 - 9 + 27 - \ldots$
9. $3 - 2 + \frac{4}{3} - \ldots$
10. $\sum_{n=1}^{\infty} 3\left(\frac{1}{4}\right)^{n-1}$
11. $\sum_{n=1}^{\infty} 5\left(-\frac{1}{10}\right)^{n-1}$
12. $\sum_{n=1}^{\infty} -\frac{2}{3}\left(-\frac{3}{4}\right)^{n-1}$

Write each repeating decimal as a fraction.

13. $0.\overline{4}$
14. $0.\overline{27}$
15. $0.\overline{123}$
16. $0.6\overline{45}$
17. $0.6\overline{7}$
18. $0.8\overline{53}$

Extra Practice

Lesson 11-6
(pages 606–610)

Find the first five terms of each sequence.

1. $a_1 = 4, a_{n+1} = 2a_n + 1$
2. $a_1 = 6, a_{n+1} = a_n + 7$
3. $a_1 = 16, a_{n+1} = a_n + (n + 4)$
4. $a_1 = 1, a_{n+1} = \frac{n}{n+2} \cdot a_n$
5. $a_1 = -\frac{1}{2}, a_{n+1} = 2a_n + \frac{1}{4}$
6. $a_1 = \frac{1}{3}, a_2 = \frac{1}{4}, a_{n+1} = a_n + a_{n-1}$

Find the first three iterates of each function for the given initial value.

7. $f(x) = 3x - 1, x_0 = 3$
8. $f(x) = 2x^2 - 8, x_0 = -1$
9. $f(x) = 4x + 5, x_0 = 0$
10. $f(x) = 3x^2 + 1, x_0 = 1$
11. $f(x) = x^2 + 4x + 4, x_0 = 1$
12. $f(x) = x^2 + 9, x_0 = 2$
13. $f(x) = 2x^2 + x + 1, x_0 = -\frac{1}{2}$
14. $f(x) = 3x^2 + 2x - 1, x_0 = \frac{2}{3}$

Lesson 11-7
(pages 612–617)

Evaluate each expression.

1. $6!$
2. $4!$
3. $\frac{13!}{6!}$
4. $\frac{10!}{3!7!}$
5. $\frac{14!}{4!10!}$
6. $\frac{7!}{2!5!}$
7. $\frac{12!}{5!}$
8. $\frac{9!}{8!}$
9. $\frac{10!}{10!0!}$

Expand each power.

10. $(z - 3)^5$
11. $(m + 1)^4$
12. $(x + 6)^4$
13. $(z - y)^2$
14. $(m + n)^5$
15. $(a - b)^4$
16. $(2n + 1)^4$
17. $(3n - 4)^3$
18. $(2n - m)^0$
19. $(4x - a)^4$
20. $(3r - 4s)^5$
21. $\left(\frac{b}{2} - 1\right)^4$

Find the indicated term of each expansion.

22. sixth term of $(x + 3)^8$
23. fourth term of $(x - 2)^7$
24. fifth term of $(a + b)^6$
25. fourth term of $(x - y)^9$
26. sixth term of $(x + 4y)^7$
27. fifth term of $(3x + 5y)^{10}$

Lesson 11-8
(pages 618–621)

Prove that each statement is true for all positive integers.

1. $2 + 4 + 6 + \ldots + 2n = n^2 + n$
2. $1^3 + 3^3 + 5^3 + \ldots + (2n - 1)^3 = n^2(2n^2 - 1)$
3. $1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \ldots + (2n - 1)(2n + 1) = \frac{n(4n^2 + 6n - 1)}{3}$
4. $\frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \ldots + \frac{1}{n(n+2)} = \frac{n(3n+5)}{4(n+1)(n+2)}$
5. $1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \ldots + n(n + 2) = \frac{n(n+1)(2n+7)}{6}$
6. $\frac{5}{1 \cdot 2} \cdot \frac{1}{3} + \frac{7}{2 \cdot 3} \cdot \frac{1}{3^2} + \frac{9}{3 \cdot 4} \cdot \frac{1}{3^3} + \ldots + \frac{2n+3}{n(n+1)} \cdot \frac{1}{3^n} = 1 - \frac{1}{3^n(n+1)}$

Find a counterexample for each statement.

7. $n^2 + 2n - 1$ is divisible by 2.
8. $2^n + 3^n$ is prime.
9. $2^{n-1} + n = 2^n + 2 - n$ for all integers $n \geq 2$.
10. $3^n - 2n = 3^n - 2^n$ for all integers $n \geq 1$

Extra Practice

Lesson 12-1

(pages 632–637)

List the possible outcomes for each situation.

1. tossing a penny and rolling a number cube
2. choosing a denim jacket that comes in dark blue, stone washed, or black that has buttons or snaps
3. ordering a large pizza with thin or thick crust, and one topping of either pepperoni, sausage, or vegetables, and either jack or mozzarella cheese

State whether the events are *independent* or *dependent*.

4. A comedy video and an action video are selected from the video store.
5. The numbers 1–10 are written on pieces of paper and are placed in a hat. Three of them are selected one after the other without replacing any of the pieces of paper.

Solve each problem.

6. On a bookshelf there are 10 different algebra books, 6 different geometry books, and 4 different calculus books. In how many ways can you choose 3 books, one of each kind?
7. In how many different ways can a 10-question true-false test be answered?
8. A student council has 6 seniors, 5 juniors, and 1 sophomore as members. In how many ways can a 3-member council committee be formed that includes one member of each class?
9. How many license plates of 5 symbols can be made using a letter for the first symbol and digits for the remaining 4 symbols?

Lesson 12-2

(pages 638–643)

Evaluate each expression.

1. $P(3, 2)$
2. $P(5, 2)$
3. $P(10, 6)$
4. $P(4, 3)$
5. $P(12, 2)$
6. $P(7, 2)$
7. $C(8, 6)$
8. $C(20, 17)$
9. $C(9, 4) \cdot C(5, 3)$
10. $C(6, 1) \cdot C(4, 1)$
11. $C(10, 5) \cdot C(8, 4)$
12. $C(7, 6) \cdot C(3, 1)$

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

13. choosing a team of 9 players from a group of 20
14. selecting the batting order of 9 players in a baseball game
15. arranging the order of 8 songs on a CD
16. finding the number of 5-card hands that include 4 diamonds and 1 club

Lesson 12-3

(pages 644–650)

A jar contains 3 red, 4 green, and 5 orange marbles. If three marbles are drawn at random and not replaced, find each probability.

1. P(all green)
2. P(1 red, then 2 not red)
3. P(2 orange, then 1 not orange)

Find the odds of an event occurring, given the probability of the event.

4. $\frac{5}{9}$
5. $\frac{4}{8}$
6. $\frac{3}{10}$

Find the probability of an event occurring, given the odds of the event.

7. $\frac{2}{7}$
8. $\frac{6}{13}$
9. $\frac{1}{19}$

The table shows the number of ways to achieve each product when two dice are tossed. Find each probability.

Product	1	2	3	4	5	6	8	9	10	12	15	16	18	20	24	25	30	36
Ways	1	2	2	3	2	4	2	1	2	4	2	1	2	2	2	1	2	1

10. $P(6)$
11. $P(12)$
12. P(not 36)
13. P(not 12)

Lesson 12-4

(pages 651–657)

An octahedral die is rolled twice. The sides are numbered 1–8. Find each probability.

1. P(1, then 8) **2.** P(two 7s) **3.** P(8, then any number)
4. P(two of the same number) **5.** P(two different numbers) **6.** P(no 8s)

Two cards are drawn from a standard deck of cards. Find each probability if no replacement occurs.

7. P(jack, jack) **8.** P(heart, club) **9.** P(two diamonds)
10. P(2 of hearts, diamond) **11.** P(2 red cards) **12.** P(2 black aces)

Determine whether the events are *independent* or *dependent*. Then find the probability.

13. According to the weather reports, the probability of rain on a certain day is 70% in Yellow Falls and 50% in Copper Creek. What is the probability that it will rain in both cities?

14. A contestant on a game show reaches into a container without looking and picks two paper bills. There are 2 \$100 bills, 4 \$50 bills, 10 \$20 bills, and 20 \$10 bills. What is the probability that the contestant draws 2 \$100 bills one after the other without replacement?

15. The odds of winning a carnival game are 1 to 5. What is the probability that a player will win the game three consecutive times?

Lesson 12-5

(pages 658–663)

An octahedral die is rolled. The sides are numbered 1–8. Find each probability.

1. P(7 or 8) **2.** P(less than 4) **3.** P(greater than 6)
4. P(not prime) **5.** P(odd or prime) **6.** P(multiple of 5 or odd)

Ten slips of paper are placed in a container. Each is labeled with a number from 1 through 10. Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

7. P(1 or 10) **8.** P(3 or odd) **9.** P(6 or less than 7)

Find each probability.

10. Two letters are chosen at random from the word GEESE and two are chosen at random from the word PLEASE. What is the probability that all four letters are Es or none of the letters is an E?

11. Three dice are thrown. What is the probability that all three dice show the same number?

12. Two marbles are simultaneously drawn at random from a bag containing 3 red, 5 blue, and 6 green marbles.
- **a.** P(at least one red marble)
- **b.** P(at least one green marble)
- **c.** P(two marbles of the same color)
- **d.** P(two marbles of different colors)

Lesson 12-6

(pages 664–670)

Find the mean, median, mode, and standard deviation of each set of data. Round to the nearest hundredth, if necessary.

1. {4, 1, 2, 1, 1} **2.** {86, 71, 74, 65, 45, 42, 76} **3.** {16, 20, 15, 14, 24, 23, 25, 10, 19}
4. {18, 24, 16, 24, 22, 24, 22, 22, 24, 13, 17, 18, 16, 20, 16, 7, 22, 5, 4, 24}
5. {55, 50, 50, 55, 65, 50, 45, 35, 50, 40, 70, 40, 70, 50, 90, 30, 35, 55, 55, 40, 75, 35, 40, 45, 65, 50, 60}
6. {364, 305, 217, 331, 305, 311, 352, 319, 272, 238, 311, 226, 220, 226, 215, 160, 123, 4, 24, 238, 99}
7. {25.5, 26.7, 20.9, 23.4, 26.8, 24.0, 25.7}
8. The following high temperatures were recorded during a cold spell in Cleveland lasting thirty-eight days.

29° 26° 17° 12° 5° 4° 25° 17° 23° 18° 13° 6° 25° 20° 27° 22° 26° 30° 31°
2° 12° 27° 16° 27° 16° 30° 6° 16° 5° 0° 5° 29° 18° 16° 22° 29° 8° 23°

Extra Practice

Lesson 12-7

(pages 671–675)

1. Determine whether the data in the table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*. The average size of a farm in each U.S. state was determined.

Acres	85–559	560–1034	1035–1509	1510–1984	1985–2459	2460–2934	2935–3409	3410–3884
States	37	4	3	1	2	1	0	2

Source: *The World Almanac*

2. The diameters of metal fittings made by a machine are normally distributed. The diameters have a mean of 7.5 centimeters and a standard deviation of 0.5 centimeters.
   a. What percent of the fittings have diameters between 7.0 and 8.0 centimeters?
   b. What percent of the fittings have diameters between 7.5 and 8.0 centimeters?
   c. What percent of the fittings have diameters greater than 6.5 centimeters?
   d. Of 100 fittings, how many will have a diameter between 6.0 and 8.5 centimeters?

3. A college entrance exam was administered at a state university. The scores were normally distributed with a mean of 510, and a standard deviation of 80.
   a. What percent would you expect to score above 510?
   b. What percent would you expect to score between 430 and 590?
   c. What is the probability that a student chosen at random scored between 350 and 670?

Lesson 12-8

(pages 676–680)

Find each probability if a coin is tossed 5 times.

1. P(0 heads)
2. P(exactly 4 heads)
3. P(exactly 3 tails)

Find each probability.

4. Ten percent of a batch of toothpaste is defective. Five tubes of toothpaste are selected at random from this batch.
   a. P(0 defective)
   b. P(exactly one defective)
   c. P(at least three defective)
   d. P(less than three defective)
5. On a 20-question true-false test, you guess at every question.
   a. P(all answers correct)
   b. P(exactly 10 correct)

Lesson 12-9

(pages 682–685)

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

1. finding the most often prescribed pain reliever by asking all of the doctors at a hospital
2. taking a poll of the most popular baby girl names this year by studying birth announcements in newspapers from different cities across the country
3. polling people who are leaving a pizza parlor about their favorite restaurant in the city

Find the margin of sampling error.

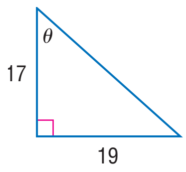
4. $p = 45\%, n = 125$
5. $p = 62\%, n = 240$
6. $p = 24\%, n = 600$
7. $p = 67\%, n = 180$
8. $p = 82\%, n = 1000$
9. $p = 15\%, n = 2500$
10. A poll conducted on the favorite breakfast choice of students in your school showed that 75% of the 2250 students asked indicated oatmeal as their favorite breakfast.
11. Of the 420 people polled at a supermarket, 56% felt that they were easily swayed by the sample items in the aisles and purchased those items, even though they were not intending to when they arrived at the store.
12. Of 3000 women between the ages of 25 and 35 polled, only 45% felt that they consume the recommended daily allowance of calcium by the National Institute of Health.

Lesson 13-1

(pages 701–708)

Find the values of the six trigonometric functions for angle θ.

1.



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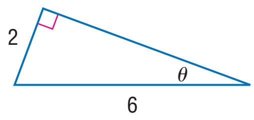
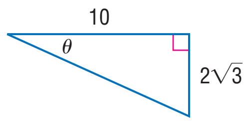
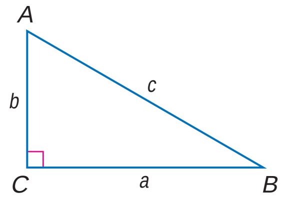


3.



Solve $\triangle ABC$ using the diagram at the right and the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



4. $B = 42°, c = 30$
5. $A = 84°, a = 4$
6. $B = 19°, b = 34$
7. $A = 75°, c = 55$
8. $b = 24, c = 36$
9. $a = 51, c = 115$
10. $\cos B = \frac{2}{5}, a = 12$
11. $\tan A = \frac{3}{2}, b = 22$

Lesson 13-2

(pages 709–715)

Draw an angle with the given measure in standard position.

1. $60°$ **2.** $250°$ **3.** $315°$ **4.** $150°$

Rewrite each degree measure in radians and each radian measure in degrees.

5. $-135°$ **6.** $-315°$ **7.** $45°$ **8.** $80°$ **9.** $24°$
10. $-54°$ **11.** $-\pi$ **12.** $\frac{9\pi}{4}$ **13.** $\frac{3\pi}{2}$ **14.** $-\frac{7\pi}{2}$
15. $\frac{9\pi}{10}$ 16. $\frac{17\pi}{30}$ **17.** $\frac{7\pi}{12}$ **18.** 1 **19.** $-2\frac{1}{3}$

Find one angle with positive measure and one angle with negative measure coterminal with each angle.

20. $50°$ **21.** $-75°$ **22.** $125°$ **23.** $-400°$ **24.** $550°$
25. 3π **26.** -2π **27.** $\frac{2\pi}{3}$ **28.** $\frac{12\pi}{5}$ **29.** 0

Lesson 13-3

(pages 717–724)

Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the given point.

1. $P(3, -4)$ **2.** $P(1, \sqrt{3})$ **3.** $P(0, -4)$ **4.** $P(-5, -5)$ **5.** $P(-\sqrt{2}, -\sqrt{2})$

Find the exact value of each trigonometric function.

6. $\cos 225°$ **7.** $\sin\left(-\frac{5\pi}{3}\right)$ **8.** $\tan \frac{7\pi}{6}$ **9.** $\tan(-300°)$ **10.** $\cos \frac{7\pi}{4}$

Suppose θ is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of θ.

11. $\cos \theta = -\frac{1}{3}$; Quadrant III
12. $\sec \theta = 2$; Quadrant IV
13. $\sin \theta = \frac{2}{3}$; Quadrant II
14. $\tan \theta = -4$; Quadrant IV
15. $\csc \theta = -5$; Quadrant III
16. $\cot \theta = -2$; Quadrant II
17. $\tan \theta = \frac{1}{3}$; Quadrant III
18. $\cos \theta = \frac{1}{4}$; Quadrant I
19. $\csc \theta = -\frac{5}{2}$; Quadrant IV

Lesson 13-4

(pages 725–732)

Find the area of $\triangle ABC$. Round to the nearest tenth.

1. $a = 11$ m, $b = 13$ m, $C = 31°$
2. $a = 15$ ft, $b = 22$ ft, $C = 90°$
3. $a = 12$ cm, $b = 12$ cm, $C = 50°$

Solve each triangle. Round to the nearest tenth.

4. $A = 18°, B = 37°, a = 15$
5. $A = 60°, C = 25°, c = 3$
6. $B = 40°, C = 32°, b = 10$
7. $B = 10°, C = 23°, c = 8$
8. $A = 12°, B = 60°, b = 5$
9. $A = 35°, C = 45°, a = 30$

Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round to the nearest tenth.

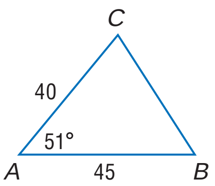
10. $A = 40°, B = 60°, c = 20$
11. $B = 70°, C = 58°, a = 84$
12. $A = 40°, a = 5, b = 12$
13. $A = 58°, a = 26, b = 29$
14. $A = 38°, B = 63°, c = 15$
15. $A = 150°, a = 6, b = 8$
16. $A = 57°, a = 12, b = 19$
17. $A = 25°, a = 125, b = 150$
18. $C = 98°, a = 64, c = 90$
19. $A = 40°, B = 60°, c = 20$
20. $A = 132°, a = 33, b = 50$
21. $A = 45°, a = 83, b = 79$

Lesson 13-5

(pages 733–738)

Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle.

1.



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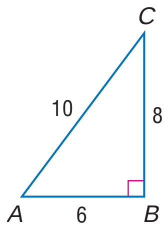
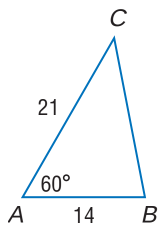


3.



4. $a = 14, b = 15, c = 16$
5. $B = 41°, C = 52°, c = 27$
6. $a = 19, b = 24.3, c = 21.8$
7. $A = 112°, a = 32, c = 20$
8. $b = 8, c = 7, A = 28°$
9. $a = 5, b = 6, c = 7$
10. $C = 25°, a = 12, b = 9$
11. $a = 8, A = 49°, B = 58°$
12. $A = 42°, b = 120, c = 160$
13. $c = 10, A = 35°, C = 65°$
14. $a = 10, b = 16, c = 19$
15. $B = 45°, a = 40, c = 48$
16. $B = 100°, a = 10, c = 8$
17. $A = 40°, B = 45°, c = 4$
18. $A = 20°, b = 100, c = 84$

Lesson 13-6

(pages 739–745)

The given point P is located on the unit circle. Find $\sin\theta$ and $\cos\theta$.

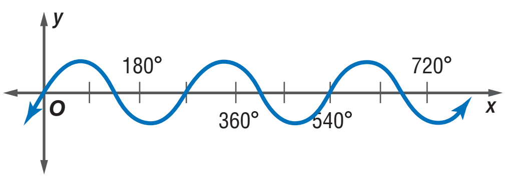
1. $P\left(\frac{4}{5}, \frac{3}{5}\right)$
2. $P\left(\frac{12}{13}, -\frac{5}{13}\right)$
3. $P\left(-\frac{8}{17}, -\frac{15}{17}\right)$
4. $P\left(\frac{3}{7}, \frac{2\sqrt{10}}{7}\right)$
5. $P\left(-\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$

Find the exact value of each function.

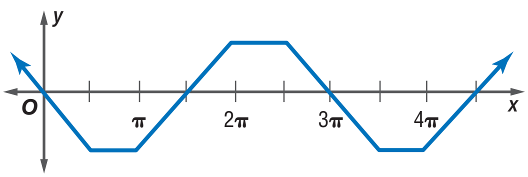
6. $\sin 210°$
7. $\cos 150°$
8. $\cos(-135°)$
9. $\cos \frac{3\pi}{4}$
10. $\sin 570°$
11. $\sin 390°$
12. $\sin \frac{4\pi}{3}$
13. $\cos\left(-\frac{7\pi}{3}\right)$
14. $\cos 30° + \cos 60°$
15. $5(\sin 45°)(\cos 45°)$
16. $\frac{\sin 210° + \cos 240°}{2}$
17. $\frac{6\cos 120° + 4\sin 150°}{5}$

Determine the period of each function.

18.



19.



Lesson 13–7

(pages 746–751)

Write each equation in the form of an inverse function.

1. $\text{Sin } m = n$
2. $\text{Tan } 45° = 1$
3. $\text{Cos } x = \frac{1}{2}$
4. $\text{Sin } 65° = a$
5. $\text{Tan } 60° = \sqrt{3}$
6. $\text{Sin } x = \frac{\sqrt{2}}{2}$

Solve each equation.

7. $y = \text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
8. $\text{Tan}^{-1}(1) = x$
9. $a = \text{Arccos}\left(\frac{\sqrt{3}}{2}\right)$
10. $\text{Arcsin}(0) = x$
11. $y = \text{Cos}^{-1}\left(\frac{1}{2}\right)$
12. $y = \text{Sin}^{-1}(1)$

Find each value. Round to the nearest hundredth.

13. $\text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)$
14. $\text{Sin}^{-1}(-1)$
15. $\cos\left[\text{Arcsin}\left(\frac{\sqrt{2}}{2}\right)\right]$
16. $\tan\left[\text{Sin}^{-1}\left(\frac{5}{13}\right)\right]$
17. $\sin 2\left[\text{Arccos}\left(\frac{1}{2}\right)\right]$
18. $\sin\left[\text{Arccos}\left(\frac{5}{17}\right)\right]$
19. $\sin\left[\text{Tan}^{-1}\left(\frac{5}{12}\right)\right]$
20. $\tan\left[\text{Arccos}\left(-\frac{\sqrt{3}}{2}\right)\right]$
21. $\sin^{-1}[\text{Cos}^{-1}(1) - 1]$
22. $\text{Cos}^{-1}\left[\tan\left(\frac{\pi}{4}\right)\right]$
23. $\cos\left[\text{Sin}^{-1}\left(\frac{1}{2}\right)\right]$
24. $\sin[\text{Cos}^{-1}(0)]$

Lesson 14-1

(pages 762–768)

Find the amplitude, if it exists, and period of each function. Then graph each function.

1. $y = 2\cos\theta$
2. $y = \frac{1}{3}\sin\theta$
3. $y = \sin 3\theta$
4. $y = 3\sec\theta$
5. $y = \sec\frac{1}{3}\theta$
6. $y = 2\csc\theta$
7. $y = 3\tan\theta$
8. $y = 3\sin\frac{2}{3}\theta$
9. $y = 2\sin\frac{1}{5}\theta$
10. $y = 3\sin 2\theta$
11. $y = \frac{1}{2}\cos\frac{3}{4}\theta$
12. $y = 5\csc 3\theta$
13. $y = 2\cot 6\theta$
14. $y = 2\csc 6\theta$
15. $y = 3\tan\frac{1}{3}\theta$

Lesson 14-2

(pages 769–776)

State the phase shift for each function. Then graph the function.

1. $y = \sin(\theta + 60°)$
2. $y = \cos(\theta - 90°)$
3. $y = \tan\left(\theta + \frac{\pi}{2}\right)$
4. $y = \sin\left(\theta + \frac{\pi}{6}\right)$

State the vertical shift and the equation of the midline for each function. Then graph the function.

5. $y = \cos\theta + 3$
6. $y = \sin\theta - 2$
7. $y = \sec\theta + 5$
8. $y = \csc\theta - 6$
9. $y = 2\sin\theta - 4$
10. $y = \frac{1}{3}\sin\theta + 7$

State the vertical shift, amplitude, period, and phase shift of each function. Then graph the function.

11. $y = 3\cos[2(\theta + 30°)] + 4$
12. $y = 2\tan[3(\theta - 60°)] - 2$
13. $y = \frac{1}{2}\sin[4(\theta - 45°)] + 1$
14. $y = \frac{2}{5}\cos[6(\theta + 45°)] - 5$
15. $y = 6 + 2\sin\left[3\left(\theta + \frac{\pi}{2}\right)\right]$
16. $y = 3 + 3\cos\left[2\left(\theta - \frac{\pi}{3}\right)\right]$

Extra Practice

Lesson 14-3

(pages 777–781)

Find the value of each expression.

1. $\sin\theta$, if $\cos\theta = \frac{4}{5}$; $0° \le \theta \le 90°$
2. $\tan\theta$, if $\sin\theta = \frac{1}{2}$; $0° \le \theta \le 90°$
3. $\csc\theta$, if $\sin\theta = \frac{3}{4}$; $90° \le \theta \le 180°$
4. $\cos\theta$, if $\tan\theta = -4$; $90° \le \theta \le 180°$
5. $\sec\theta$, if $\tan\theta = -4$; $90° \le \theta \le 180°$
6. $\sin\theta$, if $\cot\theta = -\frac{1}{4}$; $270° \le \theta \le 360°$
7. $\tan\theta$, if $\sec\theta = -3$; $90° \le \theta \le 180°$
8. $\sin\theta$, if $\cos\theta = \frac{3}{5}$; $270° \le \theta \le 360°$
9. $\cos\theta$, if $\sin\theta = -\frac{1}{2}$; $270° \le \theta \le 360°$
10. $\csc\theta$, if $\cot\theta = -\frac{1}{4}$; $90° \le \theta \le 180°$
11. $\csc\theta$, if $\sec\theta = -\frac{5}{3}$; $180° \le \theta \le 270°$
12. $\cos\theta$, if $\tan\theta = 5$; $180° \le \theta \le 270°$

Simplify each expression.

13. $\csc^2\theta - \cot^2\theta$
14. $\sin\theta\tan\theta\csc\theta$
15. $\tan\theta\csc\theta$
16. $\sec\theta\cot\theta\cos\theta$
17. $\cos\theta\,(1 - \cos^2\theta)$
18. $\frac{1 - \sin^2\theta}{\cos^2\theta}$
19. $\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$
20. $\frac{1 + \tan^2\theta}{1 + \cot^2\theta}$
21. $\frac{1}{1 + \sin\theta} + \frac{1}{1 - \sin\theta}$

Lesson 14-4

(pages 782–785)

Verify that each of the following is an identity.

1. $\sin^2\theta + \cos^2\theta + \tan^2\theta = \sec^2\theta$
2. $\frac{\tan\theta}{\sin\theta} = \sec\theta$
3. $\frac{\tan\theta}{\cot\theta} = \tan^2\theta$
4. $\csc^2\theta\,(1 - \cos^2\theta) = 1$
5. $1 - \cot^4\theta = 2\csc^2\theta - \csc^4\theta$
6. $\sin^4\theta - \cos^4\theta = \sin^2\theta - \cos^2\theta$
7. $\sin^2\theta + \cot^2\theta\sin^2\theta = 1$
8. $\frac{\cos\theta}{\csc\theta} - \frac{\csc\theta}{\sec\theta} = -\frac{\cos^3\theta}{\sin\theta}$
9. $\frac{\cos\theta}{\sec\theta - 1} + \frac{\cos\theta}{\sec\theta + 1} = 2\cot^2\theta$
10. $\frac{1 + \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 - \cos\theta}$
11. $\sec\theta + \tan\theta = \frac{\cos\theta}{1 - \sin\theta}$
12. $\tan\theta + \cot\theta = \csc\theta\sec\theta$
13. $\frac{\cot^2\theta}{1 + \cot^2\theta} = 1 - \sin^2\theta$
14. $\frac{\tan\theta - \sin\theta}{\sec\theta} = \frac{\sin^3\theta}{1 + \cos\theta}$
15. $\sin^2\theta(1 - \cos^2\theta) = \sin^4\theta$
16. $\sin^2\theta + \sin^2\theta\tan^2\theta = \tan^2\theta$
17. $\frac{\sec\theta - 1}{\sec\theta + 1} + \frac{\cos\theta - 1}{\cos\theta + 1} = 0$
18. $\tan^2\theta(1 - \sin^2\theta) = \sin^2\theta$
19. $\tan\theta + \frac{\cos\theta}{1 + \sin\theta} = \sec\theta$
20. $\frac{\tan\theta}{\sec\theta + 1} = \frac{1 - \cos\theta}{\sin\theta}$
21. $\csc\theta - \frac{\sin\theta}{1 + \cos\theta} = \cot\theta$

Lesson 14-5

(pages 786–790)

Find the exact value of each expression.

1. $\sin 195°$
2. $\cos 285°$
3. $\sin 255°$
4. $\sin 105°$
5. $\cos 15°$
6. $\sin 15°$
7. $\cos 375°$
8. $\sin 165°$
9. $\sin(-225°)$
10. $\cos(-210°)$
11. $\cos(-225°)$
12. $\sin(-30°)$
13. $\sin 120°$
14. $\sin 225°$
15. $\cos(-30°)$

Verify that each of the following is an identity.

16. $\sin(90° + \theta) = \cos\theta$
17. $\cos(180° - \theta) = -\cos\theta$
18. $\sin(\pi + \theta) = -\sin\theta$
19. $\sin(\theta + 30°) + \sin(\theta + 60°) = \frac{\sqrt{3} + 1}{2}(\sin\theta + \cos\theta)$
20. $\cos(30° - \theta) + \cos(30° + \theta) = \sqrt{3}\cos\theta$

Extra Practice

Lesson 14-6

(pages 791–797)

Find the exact value of sin 2θ, cos 2θ, sin $\frac{\theta}{2}$, and cos $\frac{\theta}{2}$ for each of the following.

1. $\cos\theta = \frac{7}{25}$; $0 < \theta < 90°$
2. $\sin\theta = \frac{2}{7}$; $0 < \theta < 90°$
3. $\cos\theta = -\frac{1}{8}$; $180 < \theta < 270°$
4. $\sin\theta = -\frac{5}{13}$; $270 < \theta < 360°$
5. $\sin\theta = \frac{\sqrt{35}}{6}$; $0 < \theta < 90°$
6. $\cos\theta = -\frac{17}{18}$; $90 < \theta < 180°$

Find the exact value of each expression by using the half-angle formulas.

7. $\sin 75°$
8. $\cos 75°$
9. $\sin\frac{\pi}{8}$
10. $\cos\frac{13\pi}{12}$
11. $\cos 22.5°$
12. $\cos\frac{\pi}{4}$

Verify that each of the following is an identity.

13. $\frac{\sin 2\theta}{2\sin^2\theta} = \cot\theta$
14. $1 + \cos 2\theta = \frac{2}{1 + \tan^2\theta}$
15. $\csc\theta\sec\theta = 2\csc 2\theta$
16. $\sin 2\theta(\cot\theta + \tan\theta) = 2$
17. $\frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \cos 2\theta$
18. $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1 + \sin 2\theta}{\cos 2\theta}$

Lesson 14-7

(pages 799–804)

Find all the solutions for each equation for $0° \leq \theta < 360°$.

1. $\cos\theta = -\frac{\sqrt{3}}{2}$
2. $\sin 2\theta = -\frac{\sqrt{3}}{2}$
3. $\cos 2\theta = 8 - 15\sin\theta$
4. $\sin\theta + \cos\theta = 1$
5. $2\sin^2\theta + \sin\theta = 0$
6. $\sin 2\theta = \cos\theta$

Solve each equation for all values of θ if θ is measured in radians.

7. $\cos 2\theta\sin\theta = 1$
8. $\sin\frac{\theta}{2} + \cos\frac{\theta}{2} = \sqrt{2}$
9. $\cos 2\theta + 4\cos\theta = -3$
10. $\sin\frac{\theta}{2} + \cos\theta = 1$
11. $3\tan^2\theta - \sqrt{3}\tan\theta = 0$
12. $4\sin\theta\cos\theta = -\sqrt{3}$

Solve each equation for all values of θ if θ is measured in degrees.

13. $2\sin^2\theta - 1 = 0$
14. $\cos\theta - 2\cos\theta\sin\theta = 0$
15. $\cos 2\theta\sin\theta = 1$
16. $(\tan\theta - 1)(2\cos\theta + 1) = 0$
17. $2\cos^2\theta = 0.5$
18. $\sin\theta\tan\theta - \tan\theta = 0$

Solve each equation for all values of θ.

19. $\tan\theta = 1$
20. $\cos 8\theta = 1$
21. $\sin\theta + 1 = \cos 2\theta$
22. $8\sin\theta\cos\theta = 2\sqrt{3}$
23. $\cos\theta = 1 + \sin\theta$
24. $2\cos^2\theta = \cos\theta$

Mixed Problem Solving

Chapter 1 Solving Equations and Inequalities (pages 4–53)

GEOMETRY **For Exercises 1 and 2, use the following information.**
The formula for the surface area of a sphere is $SA = 4\pi r^2$, and the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. *(Lesson 1-1)*

1. Find the volume and surface area of a sphere with radius 2 inches. Write your answer in terms of π.
2. Is it possible for a sphere to have the same numerical value for the surface area and volume? If so, find the radius of such a sphere.

3. **CONSTRUCTION** The Birtic family is building a family room on their house. The dimensions of the room are 26 feet by 28 feet. Show how to use the Distributive Property to mentally calculate the area of the room. *(Lesson 1-2)*

GEOMETRY **For Exercises 4–6, use the following information.**
The formula for the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$. *(Lesson 1-2)*

4. Use the Distributive Property to rewrite the formula by factoring out the greatest common factor of the two terms.
5. Find the surface area for a cylinder with radius 3 centimeters and height 10 centimeters using both formulas. Leave the answer in terms of π.
6. Which formula do you prefer? Explain your reasoning.

POPULATION **For Exercises 7 and 8, use the following information.**
In 1990, the population of Mankato, Minnesota, was 31,460. For each of the next eight years, the population decreased by an average of 85 people per year. *(Lesson 1-3)*

7. What was the population in 1998?
8. If the population continues to decline at the same rate as from 1990 to 1998, what would you expect the population to be in 2005?

9. **WEATHER** The average yearly temperature for a particular coastal city in California is 64°F. The temperature seldom varies more than 7 degrees from the average. Write and solve an equation describing the maximum and minimum temperatures for this city. *(Lesson 1-4)*

ASTRONOMY **For Exercises 10 and 11, use the following information.**
The planets in our solar system travel in orbits that are not circular. For example, Pluto's farthest distance from the Sun is 4539 million miles, and its closest distance is 2756 million miles. *(Lesson 1-4)*

10. What is the average of the two distances?
11. Write an equation that can be solved to find the minimum and maximum distances from the Sun to Pluto.

HEALTH **For Exercises 12 and 13, use the following information.**
The National Heart Association recommends that less than 30% of a person's total daily Caloric intake come from fat. One gram of fat yields nine Calories. Jason is a healthy 21-year old male whose average daily Caloric intake is between 2500 and 3300 Calories. *(Lesson 1-5)*

12. Write an inequality that represents the suggested fat intake for Jason.
13. What is the greatest suggested fat intake for Jason?

TRAVEL **For Exercises 14 and 15, use the following information.**
Bonnie is planning a 5-day trip to a convention. She wants to spend no more than $1000. The plane ticket is $375, and the hotel is $85 per night. *(Lesson 1-5)*

14. Let f represent the cost of food for one day. Write an inequality to represent this situation.
15. Solve the inequality and interpret the solution.

16. **PAINTING** Phil owns and operates a home remodeling business. He estimates that he will need 12–15 gallons of paint for a particular project. If each gallon of paint costs $18.99, write and solve a compound inequality to determine what the cost c of the paint could be. *(Lesson 1-6)*

CONSTRUCTION **For Exercises 17 and 18, use the following information.**
A new playground is to be built in the shape of a rectangle. The length must be 1.5 times the width. The playground must be more than 3750 square feet, but less than 15,000 square feet. *(Lesson 1-6)*

17. Write and solve a compound inequality to determine possible dimensions for the playground.
18. Give three possible dimensions for the playground.

Mixed Problem Solving

AGRICULTURE **For Exercises 1–3, use the following information.**
The table shows the average prices received by farmers for a bushel of corn from 1940–1999. *(Lesson 2-1)*

Year	Price	Year	Price
1940	\$0.62	1980	\$3.11
1950	\$1.52	1990	\$2.28
1960	\$1.00	1999	\$1.90
1970	\$1.33		

Source: *The World Almanac*

1. Write a relation to represent the data.
2. Graph the relation.
3. Is the relation a function? Explain your reasoning.

MEASUREMENT **For Exercises 4 and 5, use the following information.**
The equation $y = 0.3937x$ can be used to convert any number of centimeters x to inches y. *(Lesson 2-2)*

4. Find the number of inches in 100 centimeters.
5. Find the number of centimeters in 12 inches.

POPULATION **For Exercises 6–8, use the following information.**
The table shows the population of Miami, Florida, for various years since 1950. *(Lesson 2-3)*

Year	Population	Year	Population
1950	249,276	1990	358,648
1970	334,859	1998	368,624
1980	346,681	2001	365,127

Source: *The World Almanac*

6. Graph the data in the table.
7. Find the average rate of change in population from 1950 to 2001.
8. Find the rate of change from 1998 to 2001. What does your answer mean?

HEALTH **For Exercises 9–11, use the following information.**
In 1985, 39% of people in the United States age 12 and over reported using cigarettes. The percent of people using cigarettes has decreased about 1% per year following 1985. **Source:** *The World Almanac* *(Lesson 2-4)*

9. Write an equation that represents how many people use cigarettes in x years.
10. If the percent of people using cigarettes continues to decrease at the same rate, what percent of people would you predict to be using cigarettes in 2005?
11. If the trend continues, when would you predict there to be no people using cigarettes in the U.S.? How accurate is your prediction?

EMPLOYMENT **For Exercises 12–16, use the following information.**
The table shows the number of unemployed people in the United States and the percent of the population unemployed for 1993 to 1999. *(Lesson 2-5)*

Year	Number Unemployed	Percent Unemployed
1993	8,940,000	6.9
1994	7,996,000	6.1
1995	7,404,000	5.6
1996	7,236,000	5.4
1997	6,739,000	4.9
1998	6,210,000	4.5
1999	5,880,000	4.2

Source: *The World Almanac*

12. Draw two scatter plots of the data: one with year and number unemployed and the other with year and percent unemployed.
13. Use two ordered pairs to write a prediction equation for each scatter plot.
14. Compare the two equations.
15. Predict the percent of people that will be unemployed in 2005.
16. In 1999, what was the total number of people in the United States?

17. **EDUCATION** At Madison Elementary, each classroom of students can have a maximum of 25 students. Draw a graph of a step function that shows the relationship between the number of students x and the number of classrooms y that are needed. *(Lesson 2-6)*

CRAFTS **For Exercises 18–20, use the following information.**
Priscilla makes stuffed animals and plans to sell them at a local craft show. She charges \$10 for the small animals and \$15 for the large animals. To cover expenses at the craft show, she needs to sell at least \$350 worth of animals. *(Lesson 2-7)*

18. Write an inequality that describes this situation.
19. Graph the inequality.
20. If she sells 10 small and 15 large animals, will she cover her expenses?

EXERCISE **For Exercises 1–4, use the following information.**
At Everybody's Gym, you have two options for becoming a member. You can pay \$400 per year or you can pay \$150 per year plus \$5 per visit. *(Lesson 3-1)*

1. For each option, write an equation that represents the cost of belonging to the gym.
2. Graph the equations. Estimate the break-even point for the gym memberships.
3. Explain what the break-even point means.
4. If you plan to visit the gym at least once per week during the year, which option should you choose?

POPULATION **For Exercises 5–8, use the following information.**
In 1994, there were about 66.5 million men in the United States work force and 56.6 million women. Over the years, the number of men has increased by about 0.98 million per year, and the number of women has increased by about 1.08 million per year.
Source: *The World Almanac* *(Lesson 3-2)*

5. Write a system of equations that represents the number of men and women in the work force y for any number of years x.
6. Solve the system to determine the year in which the number of men and women in the work force will be the same.
7. What would be the number of men in the work force in the year in Exercise 6?
8. Do you think the solution to this system makes sense for this situation? Explain.

9. **GEOMETRY** Find the coordinates of the vertices of the parallelogram whose sides are contained in the lines whose equations are $y = 3$, $y = 7$, $y = 2x$, and $y = 2x - 13$. *(Lesson 3-2)*

EDUCATION **For Exercises 10–13, use the following information.**
Mr. Gunlikson needs to purchase equipment for his physical education classes. His budget for the year is \$4250. He decides to purchase cross-country ski equipment. He is able to find skis for \$75 per pair and boots for \$40 per pair. He knows that he should buy more boots than skis because the skis are adjustable to several sizes of boots. *(Lesson 3-3)*

10. Let y be the number of pairs of boots and x be the number of pairs of skis. Write a system of inequalities for this situation. (Remember that the number of pairs of boots and skis must be positive.)
11. Graph the region that shows how many pairs of boots and skis he can buy.
12. Give an example of three different purchases that Mr. Gunlikson can make.
13. Suppose Mr. Gunlikson wants to spend all of the money. What combination of skis and boots should he buy? Explain.

MANUFACTURING **For Exercises 14–18, use the following information.**
A shoe manufacturer makes outdoor and indoor soccer shoes. There is a two-step process for both kinds of shoes. Each pair of outdoor shoes requires 2 hours in step one, 1 hour in step two, and produces a profit of \$20. Each pair of indoor shoes requires 1 hour in step one, 3 hours in step two, and produces a profit of \$15. The company has 40 hours of labor per day available for step one and 60 hours available for step 2. *(Lesson 3-4)*

14. Let x represent the number of pairs of outdoor shoes and let y represent the number of indoor shoes that can be produced per day. Write a system of inequalities to represent the number of pairs of outdoor and indoor soccer shoes that can be produced in one day.
15. Draw the graph showing the feasible region.
16. List the coordinates of the vertices of the feasible region.
17. Write a function for the total profit on the shoes.
18. What is the maximum profit? What is the combination of shoes for this profit?

GEOMETRY **For Exercises 19–21, use the following information.**
An isosceles trapezoid has shorter base of measure a, longer base of measure c, and congruent legs of measure b. The perimeter of the trapezoid is 58 inches. The average of the bases is 19 inches and the longer base is twice the leg plus 7. *(Lesson 3-5)*

19. Write a system of three equations that represents this situation.
20. Find the lengths of the sides of the trapezoid.
21. Find the area of the trapezoid.

22. **EDUCATION** The three American universities with the greatest endowments in 2000 were Harvard, Yale, and Stanford. Their combined endowments are \$38.1 billion. Harvard had \$0.1 billion more in endowments than Yale and Stanford together. Stanford's endowments trailed Harvard's by \$10.2 billion. What were the endowments of each of these universities? *(Lesson 3-5)*

Mixed Problem Solving

AGRICULTURE **For Exercises 1 and 2, use the following information.**
In 1999, the United States produced 62,662,000 metric tons of wheat, 9,546,000 metric tons of rice, and 239,719,000 metric tons of corn. In that same year, Russia produced 30,960,000 metric tons of wheat, 444,000 metric tons of rice, and 1,070,000 metric tons of corn. **Source:** *The World Almanac* *(Lesson 4-1)*

1. Organize the data in two different matrices.
2. What are the dimensions of the matrices in Exercise 1?

LIFE EXPECTANCY **For Exercises 3–5, use the following information.**
The table shows the life expectancy for males and females for the given years. **Source:** *The World Almanac* *(Lesson 4-2)*

Year	1910	1930	1950	1970	1990
Male	48.4	58.1	65.6	67.1	71.8
Female	51.8	61.6	71.1	74.7	78.8

3. Organize all the data in a matrix.
4. Show how to organize the data in two matrices in such a way that you can find the difference between the life expectancies of males and females for the given years. Find the difference.
5. Does addition of any two matrices you can write for the data make sense? Explain.

CRAFTS **For Exercises 6 and 7, use the following information.**
Mrs. Long is selling crocheted items at a craft fair. She sells large afghans for \$60, baby blankets for \$40, doilies for \$25, and pot holders for \$5. She takes the following number of items to the fair: 12 afghans, 25 baby blankets, 45 doilies, and 50 pot holders. *(Lesson 4-3)*

6. Write an inventory matrix for the number of each item and a cost matrix for the price of each item.
7. Suppose Mrs. Long sells all of the items. Find her total income expressed as a matrix.

GEOMETRY **For Exercises 8–11, use the following information.**
A trapezoid has vertices $T(3, 3)$, $R(-1, 3)$, $A(-2, -4)$, and $P(5, -4)$. *(Lesson 4-4)*

8. Show how to use a reflection matrix to find the vertices of $TRAP$ after a reflection over the x-axis.
9. The area of a trapezoid is found by multiplying one-half the sum of the bases by the height. Find the areas of $TRAP$ and $T'R'A'P'$. How do they compare?
10. Show how to use a matrix and scalar multiplication to find the vertices of $TRAP$ after a dilation that triples the perimeter of $TRAP$.
11. Find the areas of $TRAP$ and $T'R'A'P'$ in Exercise 10. How do they compare?

AGRICULTURE **For Exercises 12 and 13, use the following information.**
A farm has a triangular plot planted with alfalfa defined by the coordinates $\left(-\frac{1}{2}, -\frac{1}{4}\right)$, $\left(\frac{1}{3}, \frac{1}{2}\right)$, and $\left(\frac{2}{3}, -\frac{1}{2}\right)$, where units are in square miles. *(Lesson 4-5)*

12. Find the area of the region in square miles.
13. One square miles equals 640 acres. To the nearest acre, how many acres are in the triangular plot?

ART **For Exercises 14 and 15, use the following information.**
Alberda sells beads to use in making Native American jewelry. Small beads sell for \$5.80 per pound, and large beads sell for \$4.60 per pound. Bernadette bought a bag of beads for \$33.00 that contained 3 times as many pounds of the small beads as the large beads. *(Lesson 4-6)*

14. Write a system of equations using the information given.
15. How many pounds of small and large beads did Bernadette buy?

MATRICES **For Exercises 16 and 17, use the following information.**
Two 2×2 inverse matrices have a sum of $\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$. The value of each entry is no less than -3 and no greater than 2. *(Lesson 4-7)*

16. Find the two matrices that satisfy the conditions.
17. Explain your method for finding the matrices.

18. **CONSTRUCTION** Alan enjoys building decks during the summer. For labor he charges \$750 to build a small deck and \$1250 to build a large deck. During the spring and summer of 2001, he built 5 more small decks than large decks. If he earned \$11,750, how many of each type of deck did he build? *(Lesson 4-8)*

Mixed Problem Solving

EDUCATION **For Exercises 1–3, use the following information.**
In 1998 in the United States, there were 2,826,146 classroom teachers and 46,534,687 students. An average of \$6662 was spent per student.
Source: *The World Almanac* *(Lesson 5-1)*

1. Write the numbers of teachers and students in scientific notation.
2. Find the number of students per teacher. Write the answer in standard notation rounded to the nearest whole number.
3. Find the total amount of money spent for students in 1998. Write the answer in both scientific and standard notation.

POPULATION **For Exercises 4–6, use the following information.**
In 2000, the population of Mexico City, Mexico, was 18,131,000, and the population of Bombay, India, was 18,066,000. It is projected that, until the year 2015, the population of Mexico City will increase at the rate of 0.4% per year and the population of Bombay will increase at the rate of 3% per year. **Source:** *The World Almanac* *(Lesson 5-2)*

4. Let r represent the rate of increase in population for each city. Write a polynomial to represent the population of each city in 2002.
5. Predict the population of each city in 2015.
6. If the projected rates are accurate, in what year will the two cities have approximately the same population?

GEOMETRY **For Exercises 7 and 8, use the following information.**
A rectangular box for a new product is designed in such a way that the three dimensions always have a particular relationship defined by the variable x. The volume of the box can be written as $6x^3 + 31x^2 + 53x + 30$, and the height is always $x + 2$. *(Lesson 5-3)*

7. What are the width and length of the box in terms of x?
8. Will the ratio of the dimensions of the box always be the same regardless of the value of x? Explain.

GEOMETRY **For Exercises 9 and 10, use the following information.**
Hero's formula for the area of a triangle is given by $A = \sqrt{s(s - a)(s - b)(s - c)}$, where a, b, and c are the lengths of the sides of the triangle and $s = 0.5(a + b + c)$. *(Lesson 5-4)*

9. Find the lengths of the sides of the triangle given in this application of Hero's formula: $A = \sqrt{s^4 - 12s^3 + 47s^2 - 60s}$.
10. What type of triangle is this?

11. **PHYSICS** The speed of sound in a liquid is $s = \sqrt{\frac{B}{d}}$, where B is known as the bulk modulus of the liquid and d is the density of the liquid. For water $B = 2.1 \cdot 10^9$ N/m^2 and $d = 10^3$ kg/m^3. Find the speed of sound in water to the nearest meter per second. *(Lesson 5-5)*

LAW ENFORCEMENT **For Exercises 12 and 13, use the following information.**
The approximate speed s in miles per hour that a car was traveling if it skidded d feet is given by the formula $s = 5.5\sqrt{kd}$, where k is the coefficient of friction. *(Lesson 5-6)*

12. For a dry concrete road, $k = 0.8$. If a car skids 110 feet on a dry concrete road, find its speed in miles per hour to the nearest whole number.
13. Another formula using the same variables is $s = 2\sqrt{5kd}$. Compare the results using the two formulas. Explain any variations in the answers.

PHYSICS **For Exercises 14–16, use the following information.**
Kepler's Third Law of planetary motion states that the square of the orbital period of any planet, in Earth years, is equal to the cube of the planet's distance from the Sun in astronomical units (AU).
Source: *The World Almanac* *(Lesson 5-7)*

14. The orbital period of Mercury is 87.97 Earth days. What is Mercury's distance from the Sun in AU?
15. Pluto's period of revolution is 247.66 Earth years. What is Pluto's distance from the Sun?
16. What is Earth's distance from the Sun in AU? Explain your result.

PHYSICS **For Exercises 17 and 18, use the following information.**
The time T in seconds that it takes a pendulum to make a complete swing back and forth is given by the formula $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum in feet and g is the acceleration due to gravity, 32 feet per second squared. *(Lesson 5-8)*

17. In Tokyo, Japan, a huge pendulum in the Shinjuku building measures 73 feet 9.75 inches. How long does it take for the pendulum to make a complete swing? **Source:** *The Guinness Book of Records*
18. A clockmaker wants to build a pendulum that takes 20 seconds to swing back and forth. How long should the pendulum be?

NUMBER THEORY **For Exercises 19 and 20, use the following information.**
Two complex conjugate numbers have a sum of 12 and a product of 40. *(Lesson 5-9)*

19. Find the two numbers.
20. Explain the method you used to find the numbers.

Mixed Problem Solving

PHYSICS **For Exercises 1–3, use the following information.**
A model rocket is shot straight up from the top of a 100-foot building at a velocity of 800 feet per second. *(Lesson 6-1)*

1. The height $h(t)$ of the model rocket t seconds after firing is given by $h(t) = -16t^2 + at + b$ where a is the initial velocity in feet per second and b is the initial height of the rocket above the ground. Write an equation for the model rocket.
2. Find the maximum height reached by the rocket and the time that the height is reached.
3. Suppose a rocket is fired from the ground (initial height is 0). Find values for a, initial velocity, and t, time, such that the rocket reaches a height of 32,000 feet at time t.

RIDES **For Exercises 4 and 5, use the following information.**
An amusement park ride carries riders to the top of a 225-foot tower. The riders then free-fall in their seats until they reach 30 feet above the ground. *(Lesson 6-2)*

4. Use the formula $h(t) = -16t^2 + h_0$, where the time t is in seconds and the initial height h_0 is in feet to find how long the riders are in free-fall.
5. Suppose the designer of the ride wants the riders to experience free-fall for 5 seconds before stopping 30 feet above the ground. What should be the height of the tower?

CONSTRUCTION **For Exercises 6 and 7, use the following information.**
Nicole's new house has a small deck that measures 6 feet by 12 feet. She would like to build a larger deck. *(Lesson 6-3)*

6. By what amount must each dimension be increased to triple the area of the original deck?
7. What are the new dimensions of the deck?

CONSTRUCTION **For Exercises 8 and 9, use the following information.**
A contractor wants to construct a rectangular pool with a length that is twice the width. He plans to build a two-meter-wide walkway around the pool. He wants the area of the walkway to equal the surface area of the pool. *(Lesson 6-4)*

8. Find the dimensions of the pool to the nearest tenth of a meter.
9. What is the surface area of the pool to the nearest square meter?

PHYSICS **For Exercises 10–12, use the following information.**
A ball is thrown vertically into the air with a velocity of 112 feet per second. The ball was released 6 feet above the ground. The height above the ground t seconds after release is modeled by the equation $h(t) = -16t^2 + 112t + 6$. *(Lesson 6-5)*

10. When will the ball reach a height of 130 feet?
11. Will the ball ever reach 250 feet? Explain your reasoning.
12. In how many seconds after its release will the ball hit the ground?

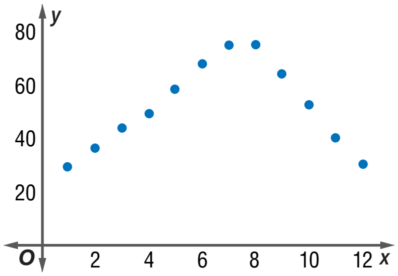
WEATHER **For Exercises 13–15, use the following information.**
The table shows the normal high temperatures for Albany, New York.
Source: *The World Almanac* *(Lesson 6-6)*

Month	Temperature (°F)
January	21
February	24
March	34
April	46
May	58
June	67
July	72
August	70
September	61
October	50
November	40
December	27

13. Suppose the months are numbered such that January = 1, February = 2, and so on. A graphing calculator gave the following function as a model for the data: $y = -1.5x^2 + 21.2x - 8.5$. Graph the points in the table and the function on the same coordinate plane.
14. Identify the vertex, axis of symmetry, and direction of opening for the calculator function.
15. Discuss how well you think the function models the actual temperature data.

16. **MODELS** John is building a display table for model cars. He wants the perimeter of the table to be 26 feet, but he wants the area of the table to be no more than 30 square feet. What could the width of the table be? *(Lesson 6-7)*

WEATHER **For Exercises 1 and 2, use the following information.**
The graph models the average monthly temperature of Boise, Idaho. The values of x are $x = 1$ for January, $x = 2$ for February, and so on. The temperatures are in °F. **Source:** *The World Almanac* *(Lesson 7-1)*



1. Is the function odd or even?
2. What do the relative maxima and minima represent?

POPULATION **For Exercises 3–5, use the following information.**
The table shows the percent of the U.S. population that was foreign-born during various years. The x values are years since 1900 and the y values are the percent of the population. **Source:** *The World Almanac* *(Lesson 7-2)*

U.S. Foreign-Born Population

x	y	x	y
0	13.6	60	5.4
10	14.7	70	4.7
20	13.2	80	6.2
30	11.6	90	8.0
40	8.8	99	9.7
50	6.9		

3. Graph the function.
4. Describe the turning points of the graph and its end behavior.
5. If this graph was modeled by a polynomial equation, what is the least degree the equation could have?

DESIGN **For Exercises 6 and 7, use the following information.**
A cylindrical can has a volume of approximately 628 cubic inches and a height of 9 inches. The volume of the can is represented by $V = \pi(a + 3)^2(9)$. *(Lesson 7-3)*

6. Use $\pi = 3.14$ to find a to the nearest tenth.
7. What is the radius of the can?

SALES **For Exercises 8 and 9, use the following information.**
The sales of items related to information technology can be modeled by $S(x) = -1.7x^3 + 18x^2 + 26.4x + 678$, where x is the number of years since 1996 and y is billions of dollars. **Source:** *Wall Street Almanac* *(Lesson 7-4)*

8. Use synthetic substitution to estimate the sales for 2003 and 2006.
9. Do you think this model is useful in estimating the sales of this industry in the future? Explain.

MANUFACTURING **For Exercises 10 and 11, use the following information.**
A box measures 12 inches by 16 inches by 18 inches. The manufacturer will increase each dimension of the box by the same number of inches and have a new volume of 5985 cubic inches. *(Lesson 7-5)*

10. Write a polynomial equation to model this situation.
11. How much should be added to each dimension?
12. **CONSTRUCTION** A picnic area has the shape of a trapezoid. The longer base is 8 more than 3 times the length of the shorter base and the height is 1 more than 3 times the shorter base. What are the dimensions of the trapezoid if the area is 4104 square feet? *(Lesson 7-6)*

EMPLOYMENT **For Exercises 13 and 14, use the following information.**
From 1994 to 1999, the number of employed women and men in the United States, age 16 and over, can be modeled by the following equations where x is the number of years since 1994 and y is the number of people in thousands. **Source:** *The World Almanac* *(Lesson 7-7)*

women: $y = 1086.4x + 56{,}610$
men: $y = 999.2x + 66{,}450$

13. Write a function that models the total number of men and women employed in the United States.
14. If f is the function for the number of men and g is the function for the number of women, what does $(f - g)(x)$ represent?
15. **HEALTH** The average weight of a baby born at a certain hospital is $7\frac{1}{2}$ pounds, and the average length is 19.5 inches. One kilogram is about 2.2 pounds, and 1 centimeter is about 0.3937 inches. Find the average weight in kilograms and the length in centimeters. *(Lesson 7-8)*

SAFETY **For Exercises 16 and 17, use the following information.**
The table shows the total stopping distance x, in meters, of a vehicle and the speed y, in meters per second. *(Lesson 7-9)*

Distance	92	68	49	32	18
Speed	29	25	20	16	11

16. Graph the data in the table.
17. Graph the function $y = 2\sqrt{2x}$ on the same coordinate plane. How well do you think this function models the given data? Explain.

CONTENTS

GEOMETRY **For Exercises 1–4, use the following information.**
Triangle *ABC* has vertices $A(2, 1)$, $B(-6, 5)$, and $C(-2, -3)$. *(Lesson 8-1)*

1. An isosceles triangle has two sides with equal length. Is triangle *ABC* isosceles? Explain your reasoning.
2. An equilateral triangle has three sides of equal length. Is triangle *ABC* equilateral? Explain your reasoning.
3. Triangle *EFG* is formed by joining the midpoints of the sides of triangle *ABC*. What type of triangle is $\triangle EFG$? Explain your reasoning.
4. Describe any relationship between the lengths of the sides of the two triangles.

ENERGY **For Exercises 5–8, use the following information.**
Solar energy plays an important role in space satellites. As they become more efficient, solar batteries are being used for more purposes on Earth. A parabolic mirror can be used to collect solar energy. The mirrors reflect the rays from the Sun to the focus of the parabola. The latus rectum of a particular mirror is 40 feet long. *(Lesson 8-2)*

5. Write an equation for the parabola formed by the mirror if the vertex of the mirror is 9.75 feet below the origin.
6. One foot is exactly 0.3048 meter. Rewrite the equation for the mirror in terms of meters.
7. Graph one of the equations for the mirror.
8. Which equation did you choose to graph? Explain.

COMMUNICATION **For Exercises 9–11, use the following information.**
Radio waves carry information from the transmitter to the receivers. The radio tower for KCGM, Voice of the Prairies, has a circular radius for broadcasting of 65 miles. The radio tower for KVCK has a circular radius for broadcasting of 85 miles. *(Lesson 8-3)*

9. Let the radio tower for KCGM be located at the origin of a coordinate system. Write an equation for the set of points at the maximum broadcast distance from the tower.
10. The radio tower for KVCK is 50 miles south and 15 miles west of the KCGM tower. Let each mile represent one unit on the coordinate system. Write an equation for the set of points at the maximum broadcast distance from the KVCK tower.
11. Graph the two equations and show the area where the radio signals overlap.

ASTRONOMY **For Exercises 12–14, use the following information.**
The table shows the closest and farthest distances of Venus and Jupiter from the Sun in millions of miles. **Source:** *The World Almanac* *(Lesson 8-4)*

Planet	Closest	Farthest
Venus	66.8	67.7
Jupiter	460.1	507.4

12. Write an equation for the orbit of each planet, assuming that the center of the orbit is the origin, the center of the Sun is a focus of the ellipse, and the Sun lies on the x-axis.
13. Find the eccentricity e, or the ratio $\frac{c}{a}$, for each planet.
14. Which planet has an orbit that is closer to looking like a circle? Explain your reasoning.
15. A comet follows a path that is one branch of a hyperbola. Suppose Earth is the center of the hyperbolic curve and has coordinates (0, 0). Write an equation for the path of the comet if $c = 5{,}225{,}000$ miles and $a = 2{,}500{,}000$ miles. Let the x-axis be the transverse axis. *(Lesson 8-5)*

AVIATION **For Exercises 16–18, use the following information.**
A military jet performs for an air show. The path of the plane during one trick can be modeled by a conic section with equation $24x^2 + 1000y - 31{,}680x - 45{,}600 = 0$, where distances are represented in feet.

16. Identify the shape of the curved path of the jet. Write the equation in standard form.
17. If the jet begins its path upward or ascent at (0, 0), what is the horizontal distance traveled by the jet from the beginning of the ascent to the end of the descent?
18. What is the maximum height of the jet?

SATELLITES **For Exercises 19–21, use the following information.**
Two satellites are placed in orbit about Earth. The equations of the two orbits are $\frac{x^2}{(300)^2} + \frac{y^2}{(900)^2} = 1$ and $\frac{x^2}{(600)^2} + \frac{y^2}{(690)^2} = 1$, where distances are in km and Earth is the center of each curve. *(Lesson 8-7)*

19. Solve each equation for y.
20. Use a graphing calculator to estimate the intersection points of the two orbits.
21. Compare the orbits of the two satellites.

Mixed Problem Solving

MANUFACTURING **For Exercises 1–3, use the following information.**
A shipping container in the shape of a rectangular prism is designed using any value for x such that the volume can be represented by the polynomial $6x^3 + 11x^2 + 4x$, where the height is x. *(Lesson 9-1)*

1. Find the length and width of the container in terms of x.
2. Find the ratio of the three dimensions of the container when $x = 2$.
3. Will the ratio of the three dimensions be the same for all values of x?

PHOTOGRAPHY **For Exercises 4–6, use the following information.**
The formula $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$ can be used to determine how far the film should be placed from the lens of a camera to create a perfect photograph. The variable q represents the distance from the lens to the film, f represents the focal length of the lens, and p represents the distance from the object to the lens. *(Lesson 9-2)*

4. Solve the formula for $\frac{1}{p}$.
5. Write the expression containing f and q as a single rational expression.
6. If a camera has a focal length of 8 centimeters and the lens is 10 centimeters from the film, how far should an object be from the lens so that the picture will be in focus?

PHYSICS **For Exercises 7–9, use the following information.**
Isaac Newton's law of universal gravitation depends upon the Inverse Square Law. It states that the relationship between two variables is related to the equation $y = \frac{1}{x^2}$. *(Lesson 9-3)*

7. Graph $y = \frac{1}{x^2}$.
8. Give the equations of any asymptotes of the graph.
9. If x represents distance in an application of the Inverse Square Law, what values of x make sense for the situation?

PHYSICS **For Exercises 10 and 11, use the following information.**
In 1798, Henry Cavendish found a value for the universal gravitational constant G that Newton used in his formula $F = G\frac{m_A m_B}{d^2}$ for finding the gravitational force between two objects. The variables in the formula are defined as follows: F is the gravitational force between the objects, G is the universal constant, m_A is the mass of the first object, m_B is the mass of the second object, and d is the distance between the centers of the objects. *(Lesson 9-4)*

10. If the mass of object A is constant, does Newton's formula represent a *direct* or *inverse* variation between the mass of object B and the distance?
11. Cavendish found the value of G to be 6.67×10^{-11} N · m²/kg². If two objects each weighing 5 kilograms are placed so that their centers are 0.5 meter apart, what is the gravitational force between the two objects?

EDUCATION **For Exercises 12–14, use the following information.**
The table shows the average number of students per computer in United States public schools for various years. *(Lesson 9-5)*

Year	Students	Year	Students
1984	125	1992	18
1985	75	1993	16
1986	50	1994	14
1987	37	1995	10.5
1988	32	1996	10
1989	25	1997	7.8
1990	22	1998	6.1
1991	20	1999	5.7

Source: *The World Almanac*

12. Let x represent years where 1984 = 1, 1985 = 2, and so on. Let y represent the number of students. Graph the data.
13. What type of function does the graph most closely resemble?
14. Use a graphing calculator to find an equation that models the data.

TRAVEL **For Exercises 15–17, use the following information.**
A trip between two towns in Montana takes 4 hours under ideal conditions. The first 150 miles of the trip is on an interstate, and the second 130 miles is on a highway with a speed limit that is 10 miles per hour less than on the interstate. *(Lesson 9-6)*

15. If x represents the speed limit on the interstate, write an expression that represents the time spent at that speed.
16. Write an expression for the time spent on the other highway.
17. Write and solve an equation to find the speed limit on the interstate and on the highway.

Mixed Problem Solving

POPULATION For Exercises 1–4, use the following information.

The population of the world has been growing rapidly. In 1950, the world population was about 2.556 billion. By 1980, it had increased to about 4.458 billion. **Source:** *The World Almanac* *(Lesson 10-1)*

1. Write an exponential function of the form $y = ab^x$ that could be used to model the world population y in billions for 1950 to 1980. Write the equation in terms of x, the number of years since 1950. (Round the value of b to the nearest ten thousandth.)
2. Suppose the world population continued to grow at that rate. Estimate the population for the year 2000.
3. In 2000, the population of the world was about 6.08 billion. Compare your estimate to the actual population.
4. Use the equation you wrote in Exercise 1 to estimate the world population in the year 2020. How accurate do you think the estimate is? Explain your reasoning.

EARTHQUAKES For Exercises 5–8, use the following information.

The table shows the Richter scale that measures earthquake intensity. Column 2 shows the increase in intensity between each number. For example, an earthquake that measures 7 is 10 times more intense than one measuring 6. *(Lesson 10-2)*

Richter Number	Increase in Magnitude
1	1
2	10
3	100
4	1000
5	10,000
6	100,000
7	1,000,000
8	10,000,000

Source: *The New York Public Library*

5. Graph this function where x is the Richter number and y is the increase in magnitude.
6. Write an equation of the form $y = b^{x-c}$ for the function in Exercise 5. (*Hint*: Write the values in the second column as powers of 10 to see a pattern and find the value of c.)
7. Graph the inverse of the function in Exercise 6.
8. Write an equation of the form $y = \log_{10}x + c$ for the function in Exercise 7.

EARTHQUAKES For Exercises 9–11, use the following information.

The table shows the magnitude on the Richter scale of some major earthquakes. *(Lesson 10-3)*

Year/Location	Magnitude
1939/Turkey	8.0
1963/Yugoslavia	6.0
1970/Peru	7.8
1988/Armenia	7.0
1995/Japan	6.9

Source: *The World Almanac*

9. Name two earthquakes such that the intensity of one was 10 times the intensity of the other.
10. Name two earthquakes such that the intensity of one was 100 times the intensity of the other.
11. What would be the magnitude of an earthquake that is 1000 times as intense as the 1963 earthquake in Yugoslavia?
12. Suppose you know that $\log_7 2 \approx 0.3562$ and $\log_7 3 \approx 0.5646$. Describe two different methods that you could use to approximate $\log_7 2.5$. (You can use a calculator, of course.) Then describe how you can check your result. *(Lesson 10-4)*

WEATHER For Exercises 13–15, use the following information.

The atmospheric pressure P, in bars, of a given height on Earth can be found by using the formula $P = s \cdot e^{-\frac{h}{H}}$. In the formula, s is the surface pressure on Earth, which is approximately 1 bar, h is the altitude for which you want to find the pressure in kilometers, and H is always 7 kilometers. *(Lesson 10-5)*

13. Find the pressure for 2 kilometers, 4 kilometers, and 7 kilometers.
14. What do you notice about the pressure as altitude increases?
15. What is the pressure on top of Mount Everest at an altitude of 8700 meters?

AGRICULTURE For Exercises 16–19, use the following information.

An equation that models the decline in the number of U.S. farms is $y = 3{,}962{,}520(0.98)^x$, where x is years since 1960 and y is the number of farms.
“**Source:** *Wall Street Journal* *(Lesson 10-6)*

16. By examining the equation, how can you determine that the number of farms is declining?
17. By what rate per year is the number of farms declining?
18. How many farms were there in 1960?
19. Predict when the number of farms will be less than 1.5 million.

Mixed Problem Solving

CLUBS **For Exercises 1 and 2, use the following information.**
Kim and her mother are in a quilting club consisting of 9 members. Each week, the club meets, and each member must bring one completed quilt square. *(Lesson 11-1)*

1. Find the first eight terms of the sequence that describes the total number of squares that have been made for the quilt after each meeting.
2. One particular quilt measures 72 inches by 84 inches and is being designed with 4-inch squares. After how many meetings will the quilt be complete?

ART **For Exercises 3 and 4, use the following information.**
Alberta is making a Native American beadwork design consisting of rows of colored beads. The first row consists of 10 beads, and each consecutive row will have 15 more beads than the previous row. *(Lesson 11-2)*

3. Write an equation for the number of beads in the nth row.
4. Find the number of beads in the design if it contains 25 rows.

GAMES **For Exercises 5 and 6, use the following information.**
An audition is held for a TV game show. At the end of each round, one-half of the prospective contestants are eliminated from the competition. On a particular day, 524 contestants begin the audition. *(Lesson 11-3)*

5. Write an equation for finding the number of contestants that are left after n rounds.
6. Using this method, will the number of contestants that are to be eliminated always be a whole number? Explain.

SPORTS **For Exercises 7–9, use the following information.**
Caitlin is training for a marathon (about 26 miles). Her trainer advises her to begin by running 2 miles. Then she is to run every other day. During each session, she is to multiply the distance she ran the previous session by one and a half. *(Lesson 11-4)*

7. Write the first five terms of a sequence describing the number of miles she is to run during consecutive training sessions.
8. When will she exceed 26 miles in one training session?
9. When will she have run at least 100 total miles?

GEOMETRY **For Exercises 10–12, use the following information.**
You can illustrate the sum of an infinite geometric series by using a square of paper. *(Lesson 11-5)*

10. Cut a square of paper at least 8 inches on a side. Let the square be one unit. Cut away one-half of the square. Call this piece Term 1. Next, cut away one-half of the remaining sheet of paper. Call this piece Term 2. Continue cutting the remaining paper in half and labeling the pieces with a term number as long as possible. List the fractions represented by the pieces.
11. If you could cut squares indefinitely, you would have an infinite series. Find the sum of the series.
12. How does the sum of the series relate to the original square of paper?

BIOLOGY **For Exercises 13–15, use the following information.**
In a particular forest, scientists are interested in how the population of wolves will change over the next two years. One mathematical model for animal population is the Verhulst population model. The formula for this model is $p_{n+1} = p_n + rp_n(1 - p_n)$, where n represents the number of time periods that have passed, p_n represents the percent of the maximum sustainable population that exists at time n, and r is the growth factor. *(Lesson 11-6)*

13. To find the population of the wolves after one year, you must evaluate the expression $p_1 = 0.45 + 1.5(0.45)(1 - 0.45)$. What is the value of the expression?
14. Explain what each number in the expression in Exercise 13 represents.
15. The current population of wolves is 165. Find the new population by multiplying 165 by the value in Exercise 13.

16. **PASCAL'S TRIANGLE** Study the first eight rows of Pascal's triangle. Write the sum of the terms in each row as a list. Make a conjecture about the sums of the rows of Pascal's triangle. *(Lesson 11-7)*

17. **NUMBER THEORY** Two statements that can be proved using mathematical induction are
$\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots + \frac{1}{3^n} = \frac{1}{2}\left(1 - \frac{1}{3^n}\right)$
and $\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots + \frac{1}{4^n} = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$.
Write and prove a conjecture involving $\frac{1}{5}$ that is similar to the two given statements. *(Lesson 11-8)*

Mixed Problem Solving

NUMBER THEORY **For Exercises 1–3, use the following information.**
According to the Rational Zero Theorem, if $\frac{p}{q}$ is a rational root, then p is a factor of the constant of the polynomial, and q is a factor of the leading coefficient. *(Lesson 12-1)*

1. What is the maximum number of possible rational roots that you may need to check for the polynomial $3x^4 - 5x^3 + 2x^2 - 7x + 10$? Explain your answer.
2. Why may you not need to check the maximum number of possible roots?
3. Are choosing the numerator and the denominator for a possible rational root independent or dependent events?

4. **GARDENING** A gardener is selecting plants for a special display. There are 15 varieties of pansies from which to choose. The gardener can only use 9 varieties in the display. How many ways can 9 varieties be chosen from the 15 varieties? *(Lesson 12-2)*

SPEED LIMITS **For Exercises 5–7, use the following information.**
In 1995, states were allowed to set their own highway speed limits. The table shows the number of states having each speed limit for their rural interstates. *(Lesson 12-3)*

Speed Limit	Number of States
55	1
65	20
70	18
75	11

Source: *The World Almanac*

5. If a state is randomly selected, what is the probability that its speed limit is 75?
6. If a state is randomly selected, what is the probability that its speed limit is 55?
7. If a state is randomly selected, what is the probability that its speed limit is 55 or greater?

8. **LOTTERIES** A lottery number for a particular lottery has seven digits which can be any digit from 0 to 9. It is advertised that the odds of winning the lottery are 1 to 10,000,000. Is this statement about the odds correct? Explain your reasoning. *(Lesson 12-4)*

For Exercises 9 and 10, use the following information.
The table shows the results of a survey of the most popular colors for luxury cars in 1999. *(Lesson 12-5)*

Color	% of cars	Color	% of cars
white	16.1	gold	7.0
silver	14.8	green	6.1
lt. brown	12.9	red	6.0
black	9.4	blue	4.9
gray	8.3	other	14.5

Source: *The World Almanac*

9. If a car sold in 1999 is randomly selected, what is the probability that it is white or silver?
10. In a parking lot of 1000 cars sold in 1999, how many cars would you expect to be blue or green?

EDUCATION **For Exercises 11–13, use the following information.**
The list of numbers given are the average scores for each state for the ACT for 1999–2000. *(Lesson 12-6)*
20.2, 21.3, 21.5, 20.3, 21.4, 21.5, 21.3, 20.6, 20.6, 19.9, 21.6, 21.4, 21.5, 21.4, 22.0, 21.6, 20.1, 19.6, 21.9, 20.7, 21.9, 21.3, 22.0, 18.7, 21.6, 21.8, 21.7, 21.5, 22.5, 20.7, 20.1, 22.2, 19.5, 21.4, 21.4, 20.8, 22.7, 21.4, 21.1, 19.3, 21.5, 20.0, 20.3, 21.5, 22.2, 20.5, 22.4, 20.2, 22.2, 21.6

11. Compare the mean and median of the data.
12. Find the standard deviation of the data. Round to the nearest hundredth.
13. Suppose the state with an average score of 20.0 incorrectly reported the results. The score for the state is actually 22.5. How are the mean and median of the data affected by this data change?
14. **HEALTH** The heights of students at Madison High School are normally distributed with a mean of 66 inches and a standard deviation of 2 inches. Of the 1080 students in the school, how many would you expect to be less than 62 inches tall? *(Lesson 12-7)*

EDUCATION **For Exercises 15 and 16, use the following information.**
A mathematics contest consists of four tests. Each test has ten multiple-choice questions with four answer options. *(Lesson 12-8)*

15. What is the probability that a student who randomly answers all questions on a test will get a score of 5 correct?
16. What is the probability that a student who randomly answers all questions on a test will get every question incorrect?
17. **SURVEY** A poll of 1750 people shows that 78% enjoy travel. Find the margin of the sampling error for the survey. *(Lesson 12-9)*

CABLE CARS **For Exercises 1 and 2, use the following information.**
The longest cable car route in the world is located in Venezuela. It begins at an altitude of 5379 feet and ends at an altitude of 15,629 feet. The ride is 8 miles long. **Source:** *The Guinness Book of Records* *(Lesson 13-1)*

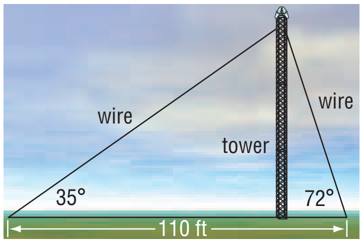
1. Draw a diagram to represent this situation.
2. To the nearest degree, what is the average angle of elevation of the cable car ride?

RIDES **For Exercises 3–5, use the following information.**
In 2000, a gigantic Ferris wheel, the London Eye, opened in England. The wheel has 32 cars evenly spaced around the circumference. *(Lesson 13-2)*

3. What is the measure, in degrees, of the angle between any two consecutive cars?
4. What is the measure, in radians, of the angle between any two consecutive cars?
5. If a car is located such that the measure in standard position is $-60°$, what are the measures of one angle with positive measure and one angle with negative measure coterminal with the angle of this car?

BASKETBALL **For Exercises 6 and 7, use the following information.**
During the halftime of a basketball game, a person is selected to try to make a shot at a distance of 12 feet from the basket. The formula $R = \frac{V_0^2 \sin 2\theta}{32}$ gives the distance of a basketball shot with an initial velocity of V_0 feet per second at an angle of θ with the ground. *(Lesson 13-3)*

6. If the basketball was shot with an initial velocity of 24 feet per second at an angle of 75°, how far will the basketball travel?
7. Find an initial velocity and angle of release that would result in the ball traveling approximately 12 feet to the basket.
8. **COMMUNICATIONS** A telecommunications tower needs to be supported by two wires. The angle between the ground and the tower on one side must be 35° and the angle between the ground and the second tower must be 72°. The distance between the two wires is 110 feet.



To the nearest foot, what should be the lengths of the two wires? *(Lesson 13-4)*

SURVEYING **For Exercises 9–11, use the following information.**
A triangular plot of farm land measures 0.9 by 0.5 by 1.25 miles. *(Lesson 13-5)*

9. If the plot of land is fenced on the border, what will be the angles at which the fences of the three sides meet? Round to the nearest degree.
10. What is the area of the plot of land? (*Hint*: Use the area formula in Lesson 13-4.)
11. One square mile is 640 acres. How many acres are in the plot of land?

WEATHER **For Exercises 12 and 13, use the following information.**
The monthly normal temperatures, in degrees Fahrenheit, for New York City are given in the table. January is assigned a value of 1, February a value of 2, and so on. *(Lesson 13-6)*

Month	Temperature	Month	Temperature
1	32	7	77
2	34	8	76
3	42	9	68
4	53	10	58
5	63	11	48
6	72	12	37

Source: *The World Almanac*

12. Graph the data in a scatter plot.
13. A trigonometric model for the temperature T in degrees Fahrenheit of New York City at t months is given by $T = 22.5 \sin\left(\frac{\pi}{6}x - 2.25\right) + 54.3$. A quadratic model for the same situation is $T = -1.34x^2 + 18.84x + 5$. Which model do you think best fits the data? Explain your reasoning.

PHYSICS **For Exercises 14–16, use the following information.**
When light passes from one substance to another, it may be reflected and refracted. Snell's law can be used to find the angle of refraction as a beam of light passes from one substance to another. One form of the formula for Snell's law is $n_1 \sin \theta_1 = n_2 \sin \theta_2$, where n_1 and n_2 are the indices of refraction for the two substances and θ_1 and θ_2 are the angles of the light rays passing through the two substances. *(Lesson 13-7)*

14. Solve the equation for $\sin \theta_1$.
15. Write an equation in the form of an inverse function that allows you to find θ_1.
16. If a light beam in air with index of refraction of 1.00 hits a diamond with index of 2.42 at an angle of 30°, find the angle of refraction.

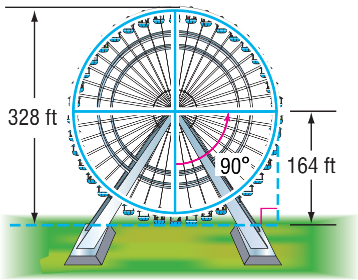
Mixed Problem Solving

CONTENTS

TIDES **For Exercises 1–3, use the following information.**
The world's record for the highest tide is held by the Minas Basin in Nova Scotia, Canada, with a tidal range of 54.6 feet. A tide is at equilibrium when it is at its normal level halfway between its highest and lowest points. *(Lesson 14-1)*

1. Write an equation to represent the height h of the tide. Assume that the tide is at equilibrium at $t = 0$, that the high tide is beginning, and that the tide completes one cycle in 12 hours.
2. Mobile, Alabama, has a very small tidal range at only one foot six inches. Write an equation to represent the height h of the tide in Mobile.
3. Graph the functions for the tides in Minas Basin and Mobile on the same axis system. How do the graphs compare?

RIDES **For Exercises 4–7, use the following information.**
The Cosmoclock 21 is a huge Ferris wheel in Yokohama City, Japan. The diameter is 328 feet. Suppose that a rider enters the ride at 0 feet and then rotates in 90° increments counterclockwise. The table shows the angle measures of rotation and the height above the ground of the rider. *(Lesson 14-2)*

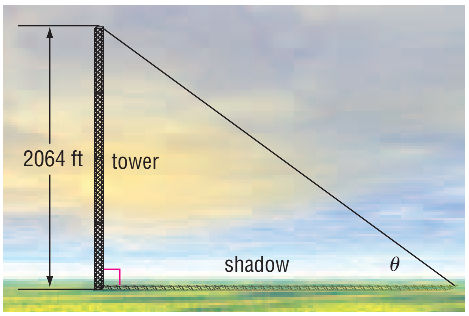


Angle	Height	Angle	Height
0°	0	450°	
90°	164	540°	
180°	328	630°	
270°		720°	
360°			

4. Copy and complete the table. Then graph the points (angle, height).
5. A function that models the data is $y = 164 \cdot (\sin(x - 90°)) + 164$. Identify the vertical shift, amplitude, period, and phase shift of the graph.
6. Write an equation using the sine that models the position of a rider on the Vienna Giant Ferris Wheel in Vienna, Austria, with a diameter of 200 feet. Check your equation by plotting the points and the equation with a graphing calculator.
7. Write an equation using the cosine that models the position of a rider on the Vienna Giant Ferris Wheel.
8. **TRIGONOMETRY** Using the exact values for the sine and cosine functions, show that the identity $\cos^2\theta + \sin^2\theta = 1$ is true for angles of measure 30°, 45°, 60°, 90°, and 180°. *(Lesson 14-3)*
9. **ROCKETS** In the formula $h = \frac{v^2 \sin^2\theta}{2g}$, h is the maximum height reached by a rocket, θ is the angle between the ground and the initial path of the object, v is the rocket's initial velocity, and g is the acceleration due to gravity. Verify the identity $\frac{v^2 \sin^2\theta}{2g} = \frac{v^2\cos^2\theta}{2g\cot^2\theta}$. *(Lesson 14-4)*

WEATHER **For Exercises 10–12, use the following information.**
The monthly high temperatures for Minneapolis, Minnesota, can be modeled by the equation $y = 31.65 \sin\left(\frac{\pi}{6}x - 2.09\right) + 52.35$, where the months x are January $= 1$, February $= 2$, and so on. The monthly low temperatures for Minneapolis can be modeled by the equation $y = 30.15 \sin\left(\frac{\pi}{6}x - 2.09\right) + 32.95$. *(Lesson 14-5)*

10. Write a new function by adding the expressions on the right side of each equation and dividing the result by 2.
11. Graph the two original functions and the new function on the same calculator screen. What do you notice?
12. What is the meaning of the function you wrote in Exercise 10?
13. Begin with one of the Pythagorean Identities. Perform equivalent operations on each side to create a new trigonometric identity. Then show that the identity is true. *(Lesson 14-6)*
14. **TELEVISION** The tallest structure in the world is a television transmitting tower located near Fargo, North Dakota, with a height of 2064 feet.



What is the measure of θ if the length of the shadow is 1 mile? **Source:** *The Guinness Book of Records* *(Lesson 14–7)*

Mixed Problem Solving

Becoming a Better Test-Taker

At some time in your life, you will probably have to take a standardized test. Sometimes this test may determine if you go on to the next grade or course, or even if you will graduate from high school. This section of your textbook is dedicated to making you a better test-taker.

TYPES OF TEST QUESTIONS In the following pages, you will see examples of four types of questions commonly seen on standardized tests. A description of each type of question is shown in the table below.

Type of Question	Description	See Pages
multiple choice	Four or five possible answer choices are given from which you choose the best answer.	878–879
gridded response	You solve the problem. Then you enter the answer in a special grid and shade in the corresponding circles.	880–883
short response	You solve the problem, showing your work and/or explaining your reasoning.	884–887
extended response	You solve a multi-part problem, showing yourwork and/or explaining your reasoning	888–892

PRACTICE After being introduced to each type of question, you can practice that type of question. Each set of practice questions is divided into five sections that represent the concepts most commonly assessed on standardized tests.

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

USING A CALCULATOR On some tests, you are permitted to use a calculator. You should check with your teacher to determine if calculator use is permitted on the test you will be taking, and if so, what type of calculator can be used.

TEST-TAKING TIPS In addition to Test-Taking Tips like the one shown at the right, here are some additional thoughts that might help you.

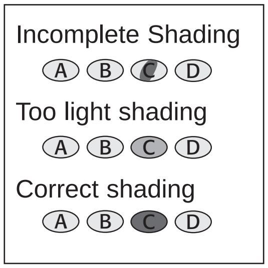
- Get a good night's rest before the test. Cramming the night before does not improve your results.
- Budget your time when taking a test. Don't dwell on problems that you cannot solve. Just make sure to leave that question blank on your answer sheet.
- Watch for key words like NOT and EXCEPT. Also look for order words like LEAST, GREATEST, FIRST, and LAST.

Test-Taking Tip

If you are allowed to use a calculator, make sure you are familiar with how it works so that you won't waste time trying to figure out the calculator when taking the test.

Multiple-Choice Questions

Multiple-choice questions are the most common type of questions on standardized tests. These questions are sometimes called *selected-response questions.* You are asked to choose the best answer from four or five possible answers. To record a multiple-choice answer, you may be asked to shade in a bubble that is a circle or an oval or to just write the letter of your choice. Always make sure that your shading is dark enough and completely covers the bubble.



The answer to a multiple-choice question is usually not immediately obvious from the choices, but you may be able to eliminate some of the possibilities by using your knowledge of mathematics. Another answer choice might be that the correct answer is not given.

Example 1

White chocolate pieces sell for $3.25 per pound while dark chocolate pieces sell for $2.50 per pound. How many pounds of white chocolate are needed to produce a 10-pound mixture of both kinds that sells for $2.80 per pound?

Strategy

Reasonableness
Check to see that your answer is reasonable with the given information.

(A) **2 lb** **(B)** **4 lb** **(C)** **6 lb** **(D)** **10 lb**

The question asks you to find the number of pounds of the white chocolate. Let w be the number of pounds of white chocolate and let d be the number of pounds of dark chocolate. Write a system of equations.

$w + d = 10$ There is a total of 10 pounds of chocolate.

$3.25w + 2.50d = 2.80(10)$ The price is $2.80 × 10 for the mixed chocolate.

Use substitution to solve.

$3.25w + 2.50d = 2.80(10)$ Original equation

$3.25w + 2.50(10 - w) = 28$ Solve the first equation for d and substitute.

$3.25w + 25 - 2.5w = 28$ Distributive Property

$0.75w = 3$ Simplify.

$w = 4$ Divide each side by 0.75.

The answer is B.

Example 2

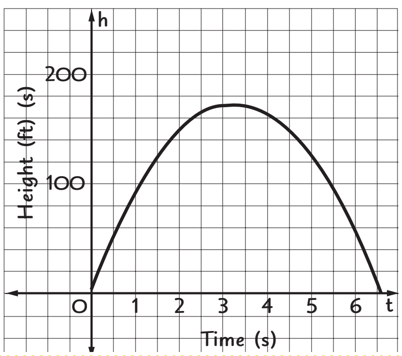
Josh throws a baseball upward at a velocity of 105 feet per second, releasing the baseball when it is 5 feet above the ground. The height of the baseball t seconds after being thrown is given by the formula $h(t) = -16t^2 + 105t + 5$. Find the time at which the baseball reaches its maximum height. Round to the nearest tenth of a second.

Strategy

Diagrams
Drawing a diagram for a situation may help you to answer the question.

(F) **1.0 s** **(G)** **3.3 s** **(H)** **6.6 s** **(J)** **177.3 s**

Graph the equation. The graph is a parabola. Make sure to label the horizontal axis as t (time in seconds) and the vertical axis as h for height in feet. The ball is at its maximum height at the vertex of the graph.



The graph indicates that the maximum height is achieved between 3 and 4 seconds after launch.

The answer is G.

Preparing for Standardized Tests

Multiple-Choice Practice

Choose the best answer.

Number and Operations

1. In 2002, 1.8123×10^8 people in the United States and Canada used the Internet while 5.442×10^8 people worldwide used the Internet. What percent of users were from the United States and Canada?

Ⓐ 33% Ⓑ 35% Ⓒ 37% Ⓓ 50%

2. Serena has 6 plants to put in her garden. How many different ways can she arrange the plants?

Ⓐ 21 Ⓑ 30 Ⓒ 360 Ⓓ 720

Algebra

3. The sum of Kevin's, Anna's, and Tia's ages is 40. Anna is 1 year more than twice as old as Tia. Kevin is 3 years older than Anna. How old is Anna?

Ⓐ 7 Ⓑ 14 Ⓒ 15 Ⓓ 18

4. Rafael's Theatre Company sells tickets for \$10. At this price, they sell 400 tickets. Rafael estimates that they would sell 40 less tickets for each \$2 price increase. What charge would give the most income?

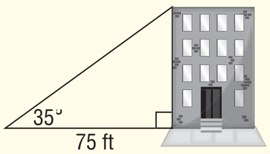
Ⓐ 10 Ⓑ 13 Ⓒ 15 Ⓓ 20

Geometry

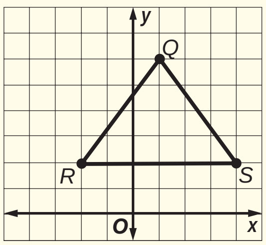
5. Hai stands 75 feet from the base of a building and sights the top at a 35° angle. What is the height of the building to the nearest tenth of a foot?

Ⓐ 0.0 ft Ⓑ 43.0 ft

Ⓒ 52.5 ft Ⓓ 61.4 ft



6. Samone draws $\triangle QRS$ on grid paper to use for a design in her art class. She needs to rotate the triangle 180° counterclockwise. What will be the y-coordinate of the image of S?



Ⓐ −6 Ⓑ −2 Ⓒ −1 Ⓓ 2

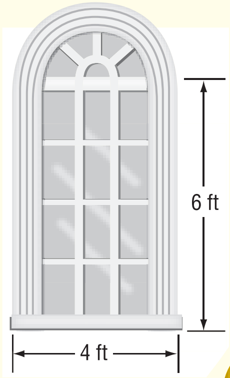
Measurement

7. Lakeisha is teaching a summer art class for children. For one project, she estimates that she will need $\frac{2}{3}$ yard of string for each 3 students. How many yards will she need for 16 students?

Ⓐ 3 yd Ⓑ $3\frac{5}{9}$ yd

Ⓒ $10\frac{2}{3}$ yd Ⓓ 16 yd

8. Kari works at night so she needs to make her room as dark as possible during the day to sleep. How much black paper will she need to cover the window in her room, which is shaped as shown. Use 3.14 for π. Round to the nearest tenth of a square foot.



Ⓐ 24.0 ft² Ⓑ 24.6 ft²

Ⓒ 30.3 ft² Ⓓ 36.6 ft²

Data Analysis and Probability

9. A card is drawn from a standard deck of 52 cards. If one card is drawn, what is the probability that it is a heart or a 2?

Ⓐ $\frac{1}{52}$ Ⓑ $\frac{1}{13}$ Ⓒ $\frac{1}{4}$ Ⓓ $\frac{4}{13}$

10. The weight of candy in boxes is normally distributed with a mean of 12 ounces and a standard deviation of 0.5 ounce. About what percent of the time will you get a box that weighs over 12.5 ounces?

Ⓐ 13.5% Ⓑ 16% Ⓒ 50% Ⓓ 68%

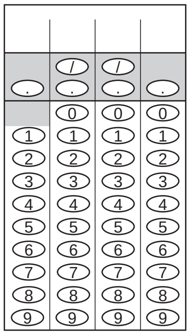
Test-Taking Tip

Question 8

Many standardized tests include a reference sheet with common formulas that you may use during the test. If it is available before the test, familiarize yourself with the reference sheet for quick reference during the test.

Preparing for Standardized Tests

Gridded-Response Questions



Gridded-response questions are another type of question on standardized tests. These questions are sometimes called *student-produced response* or *grid-in*, because you must create the answer yourself, not just choose from four or five possible answers.

For gridded response, you must mark your answer on a grid printed on an answer sheet. The grid contains a row of four or five boxes at the top, two rows of ovals or circles with decimal and fraction symbols, and four or five columns of ovals, numbered 0–9. At the right is an example of a grid from an answer sheet.

Example 1 **Find the x-coordinate for the solution of the given system of equations.**

$$4x - y = 14$$
$$-3x + y = -11$$

What value do you need to find?

You need to find only the x-coordinate of the point where the graphs of the two equations intersect. You could graph the system, but that takes time. The easiest method is probably the substitution method since the second equation can be solved easily for y.

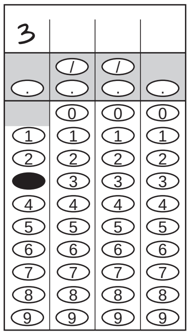
$-3x + y = -11$	Second equation
$y = -11 + 3x$	Solve the second equation for y.

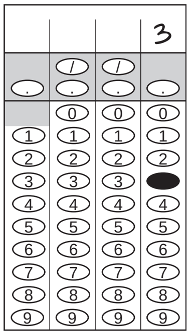
$4x - y = 14$	First equation
$4x - (-11 + 3x) = 14$	Substitute for y.
$4x + 11 - 3x = 14$	Distributive Property
$x = 3$	Simplify.

The answer to be filled in on the grid is 3.

How do you fill in the grid for the answer?

- Write your answer in the answer boxes.
- Write only one digit or symbol in each answer box.
- Do not write any digits or symbols outside the answer boxes.
- You may write your answer with the first digit in the left answer box, or with the last digit in the right answer box. You may leave blank any boxes you do not need on the right or the left side of your answer.





- Fill in only one bubble for every answer box that you have written in. Be sure not to fill in a bubble under a blank answer box.

Many gridded response questions result in an answer that is a fraction or a decimal. These values can also be filled in on the grid.

Example 2 **Zuri is solving a problem about the area of a room. The equation she needs to solve is $4x^2 + 11x - 3 = 0$. Since the answer will be a length, she only needs to find the positive root. What is the solution?**

Since you can see the equation is not easily factorable, use the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

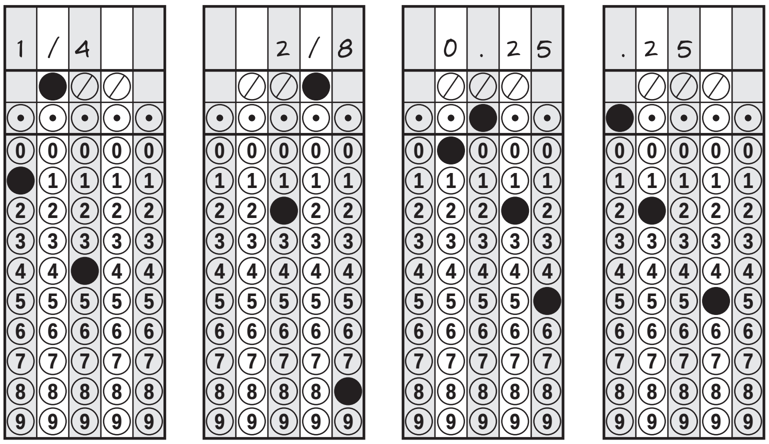
$$= \frac{-11 \pm \sqrt{11^2 - 4(4)(-3)}}{2(4)}$$

$$= \frac{-11 + 13}{8}$$ There are two roots, but you only need the positive one.

$$= \frac{2}{8} \text{ or } \frac{1}{4}$$

How do you grid the answer?

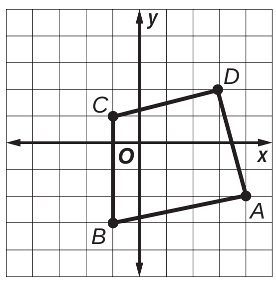
You can either grid the fraction $\frac{1}{4}$, or rewrite it as 0.25 and grid the decimal. Be sure to write the decimal point or fraction bar in the answer box. The following are acceptable answer responses that represent $\frac{1}{4}$ and 0.25.



Do not leave a blank answer box in the middle of an answer.

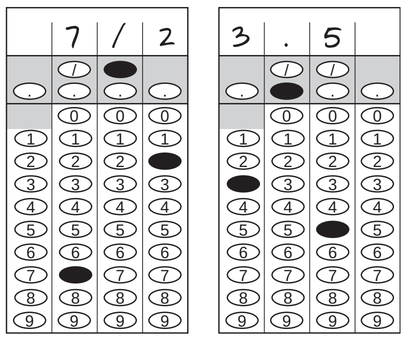
Some problems may result in an answer that is a mixed number. Before filling in the grid, change the mixed number to an equivalent improper fraction or decimal. For example, if the answer is $1\frac{1}{2}$, do not enter 1 1/2 as this will be interpreted as $\frac{11}{2}$. Instead, either enter 3/2 or 1.5.

Example 3 **José is using this figure for a computer graphics design. He wants to dilate the figure by a scale factor of $\frac{7}{4}$. What will be the y-coordinate of the image of D?**



To find the y-coordinate of the image of D, multiply the x-coordinate by the scale factor of $\frac{7}{4}$.

$$2 \cdot \frac{7}{4} = \frac{7}{2}$$



Grid in 7/2 or 3.5. Do not grid 3 1/2.

Gridded-Response Practice

Solve each problem. Then copy and complete a grid.

Number and Operations

1. Rewrite 16^3 as a power of 2. What is the value of the exponent for 2?

2. Wolf 359 is the fourth closest star to Earth. It is 45,531,250 million miles from Earth. A light-year, the distance light travels in a year, is 5.88×10^{12} miles. What is the distance from Earth to Wolf 359 in light-years? Round to the nearest tenth.

3. A store received a shipment of coats. The coats were marked up 50% to sell to the customers. At the end of the season, the coats were discounted 60%. Find the ratio of the discounted price of a coat to the original cost of the coat to the store.

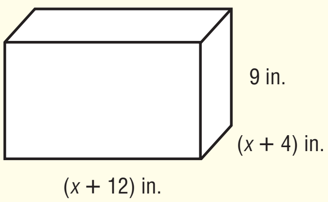
4. Find the value of the determinant $\begin{vmatrix} -1 & 4 \\ -3 & 0 \end{vmatrix}$.

5. Kendra is displaying eight sweaters in a store window. There are four identical red sweaters, three identical brown sweaters, and one white sweater. How many different arrangements of the eight sweaters are possible?

6. An electronics store reduced the price of a DVD player by 10% because it was used as a display model. If the reduced price was \$107.10, what was the cost in dollars before it was reduced? Round to the nearest cent if necessary.

Algebra

7. The box shown can be purchased to ship merchandise at the Pack 'n Ship Store. The volume of the box is 945 cubic inches. What is the measure of the greatest dimension of the box in inches?



8. If $f(x) = 2x^2 - 3x + 10$, find $f(-1)$.

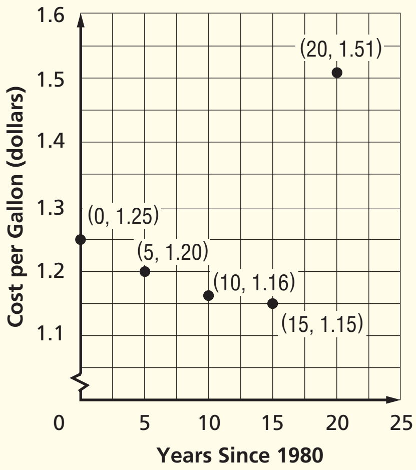
Test-Taking Tip

Question 22
Fractions do not have to be written in lowest terms. Any equivalent fraction that fits the grid is correct.

9. Find the positive root of $\frac{10}{a^2 - 16} - \frac{6}{a + 4} = \frac{4}{9}$.

10. The graph shows the retail price per gallon of unleaded gasoline in the U.S. from 1980 to 2000. What is the slope if a line is drawn through the points for 15 and 20 years since 1980?

Retail Price of Unleaded Gasoline

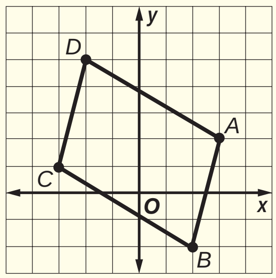


Source: U.S. Dept. of Energy

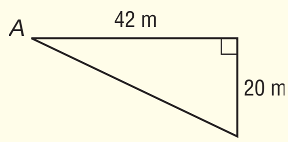
11. Solve $\sqrt{x + 11} - 9 = \sqrt{x} - 8$.

Geometry

12. Polygon $DABC$ is rotated 90° counterclockwise and then reflected over the line $y = x$. What is the x-coordinate of the final image of A?



13. A garden is shaped as shown below. What is the measure of $\angle A$ to the nearest degree?



14. A circle of radius r is circumscribed about a square. What is the ratio of the area of the circle to the area of the square? Express the ratio as a decimal rounded to the nearest hundredth.

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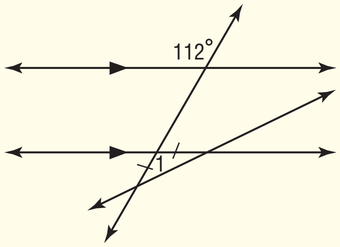
15. Find the degree measure of $\angle 1$.



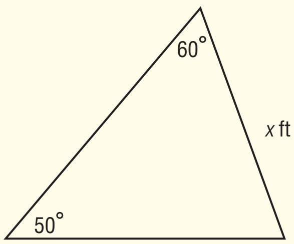
16. Each angle of a regular polygon measures 150°. How many sides does the polygon have?

Measurement

17. The Pasacal (P) is a measure of pressure that is equivalent to 1 Newton per square meter. The typical pressure in an automobile tire is 2×10^5 P while typical blood pressure is 1.6×10^4 P. How many times greater is the pressure in a tire than typical blood pressure?

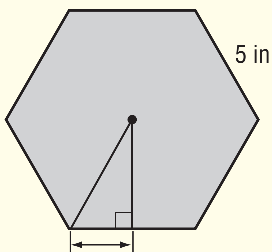
18. A circular ride at an amusement park rotated $\frac{7\pi}{4}$ radians while loading riders. What is the degree measure of the rotation?

19. Find the value of x in the triangle. Round to the nearest tenth of a foot.



20. Four equal-sized cylindrical juice cans are packed tightly in the box shown. What is the volume of space in the box that is not occupied by the cans in cubic inches? Use 3.14 for π and round to the nearest cubic inch.

21. Caroline is making a quilt. The diagram shows a piece of cloth she will cut for a portion of the pattern. Find the area of the entire hexagonal piece to the nearest tenth of a square inch.



Data Analysis and Probability

22. Of ten girls on a team, three have blue eyes. If two girls are chosen at random, what is the probability that neither has blue eyes?

23. In order to win a game, Miguel needs to advance his game piece 4 spaces. What is the probability that the sum of the numbers on the two dice he rolls will be 4?

24. The table shows the number of televisions owned per 1000 people in each country. What is the absolute value of the difference between the mean and the median of the data?

Country	Televisions
United States	844
Latvia	741
Japan	719
Canada	715
Australia	706
United Kingdom	652
Norway	648
Finland	643
France	623

Source: International Telecommunication Union

25. The table shows the amount of breakfast cereal eaten per person each year by the ten countries that eat the most. Find the standard deviation of the data set. Round to the nearest tenth of a pound.

Country	Cereal (lb)
Sweden	23
Canada	17
Australia	16
United Kingdom	15
Nauru	14
New Zealand	14
Ireland	12
United States	11
Finland	10
Denmark	7

Source: Euromonitor

26. Two number cubes are rolled. If the two numbers appearing on the faces of the number cubes are different, find the probability that the sum is 6. Round to the nearest hundredth.

Short-Response Questions

Short-response questions require you to provide a solution to the problem, as well as any method, explanation, and/or justification you used to arrive at the solution. These are sometimes called *constructed-response, open-response, open-ended, free-response*, or *student-produced questions.* The following is a sample rubric, or scoring guide, for scoring short-response questions.

Criteria	Score
Full credit: The answer is correct and a full explanation is provided that shows each step in arriving at the final answer.	2
Partial credit: There are two different ways to receive partial credit. • The answer is correct, but the explanation provided is incomplete or incorrect. • The answer is incorrect, but the explanation and method of solving the problem is correct.	1
No credit: Either an answer is not provided or the answer does not make sense.	0

On some standardized tests, no credit is given for a correct answer if your work is not shown.

Example

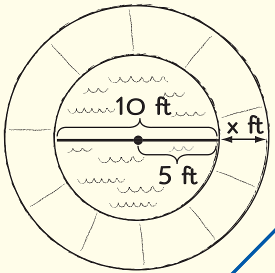
Mr. Youngblood has a fish pond in his backyard. It is circular with a diameter of 10 feet. He wants to build a walkway of equal width around the pond. He wants the total area of the pond and walkway to be about 201 square feet. To the nearest foot, what should be the width of the walkway?

Strategy

Diagrams
Draw a diagram of the pond and the walkway. Label important information.

Full Credit Solution

First draw a diagram to represent the situation.



Since the diameter of the pond is 10 feet, the radius is 5 ft. Let the width of the walkway be x feet.

$A = \pi r^2$

$201 = \pi(x + 5)^2$ $r = x + 5, A = 201$

$201 = \pi(x^2 + 10x + 25)$ Multiply.

$\frac{201}{\pi} = \frac{\pi(x^2 + 10x + 25)}{\pi}$ Divide by π, using 3.14 for π.

$64 = x^2 + 10x + 25$ or $x^2 + 10x - 39 = 0$

Use the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-10 \pm \sqrt{(-10)^2 - 4(1)(-39)}}{2(1)}$ $a = 1, b = 10, c = -39$

$= \frac{-10 + \sqrt{256}}{2}$ or 3

The width of the walkway should be 3 feet.

The steps, calculations, and reasoning are clearly stated.

Before taking a standardized test, memorize common formulas, like the Quadratic Formula, to save time.

Since length must be positive, eliminate the negative solution.

Partial Credit Solution

In this sample solution, the equation that can be used to solve the problem is correct. However, there is no justification for any of the calculations.

> There is no explanation of how the quadratic equation was found.

$x^2 + 10x - 39 = 0$

$x = \dfrac{-10 + \sqrt{256}}{2}$

$= -13 \text{ or } 3$

The walkway should be 3 feet wide.

Partial Credit Solution

In this sample solution, the answer is incorrect because the wrong root was chosen.

Since the diameter of the pond is 10 feet, the radius is 5 ft. Let the width of the walkway be x feet. Use the formula for the area of a circle.

$A = \pi r^2$

$201 = \pi(x + 5)^2$

$201 = \pi(x^2 + 10x + 25)$

$\dfrac{201}{\pi} = \dfrac{\pi(x^2 + 10x + 25)}{\pi}$

$64 = x^2 \text{ a } 10x \text{ a } 25 \text{ or } x^2 \text{ a } 10x - 39 = 0$

Use the Quadratic Formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= -13 \text{ or } 3$

The walkway should be 13 feet wide.

> The negative root was chosen as the solution.

No Credit Solution

Use the formula for the area of a circle.

$A = \pi r^2 x$

$201 = \pi(5)^2 x$

$201 = 3.14(25)x$

$201 = 78.5x$

$x = 2.56$

Build the walkway 3 feet wide.

> The width of the walkway *x* is used incorrectly in the area formula for a circle. However, when the student rounds the value for the width of the walkway, the answer is correct. No credit is given for an answer achieved using faulty reasoning.

Short-Response Practice

Solve each problem. Show all your work.

Number and Operations

1. An earthquake that measures a value of 1 on the Richter scale releases the same amount of energy as 170 grams of TNT, while one that measures 4 on the scale releases the energy of 5 metric tons of TNT. One metric ton is 1000 kilograms and 1000 grams is 1 kilograms. How many times more energy is released by an earthquake measuring 4 than one measuring 1?

2. In 2000, Cook County, Illinois was the second largest county in the U.S with a population of about 5,377,000. This was about 43.3% of the population of Illinois. What was the approximate population of Illinois in 2000?

3. Show why $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity matrix for multiplication for 2×2 matrices.

4. The total volume of the oceans on Earth is 3.24×10^8 cubic miles. The total surface area of the water of the oceans is 139.8 million square miles. What is the average depth of the oceans?

5. At the Blaine County Fair, there are 12 finalists in the technology project competition. How many ways can 1st, 2nd, 3rd, and 4th place be awarded?

Algebra

6. Factor $3x^2a^2 - 3x^2b^2$. Explain each step.

7. Solve and graph $7 - 2a > \frac{15 - 2a}{6}$.

8. Solve the system of equations.
$x^2 + 9y^2 = 25 \quad y - x = -5$

9. The table shows what Miranda Richards charges for landscaping services for various numbers of hours. Write an equation to find the charge for any amount of time, where y is the total charge in dollars and x is the amount of time in hours. Explain the meaning of the slope and y-intercept of the graph of the equation.

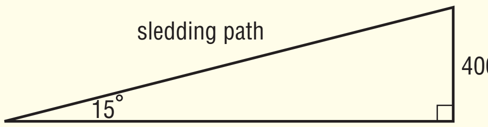
Hours	Charge (dollars)	Hours	Charge (dollars)
0	17.50	3	64.00
1	33.00	4	79.50
2	48.50	5	95.00

10. Write an equation that fits the data in the table.

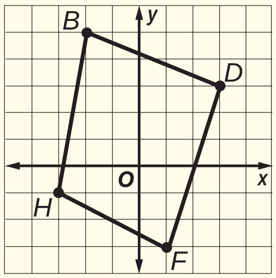
x	y
−3	12
−1	4
0	3
2	7
4	19

Geometry

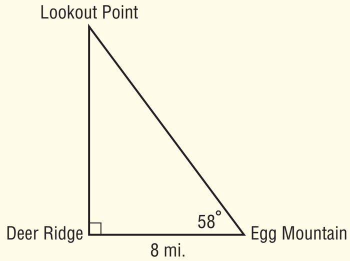
11. A sledding hill at the local park has an angle of elevation of 15°. Its vertical drop is 400 feet. What is the length of the sledding path?



12. Polygon $BDFH$ is transformed using the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Graph $B'D'F'H'$ and identify the type of transformation.



13. The map shows the trails that connect three hiking destinations. If Amparo hikes from Deer Ridge to Egg Mountain to Lookout Point and back to Deer Ridge, what is the distance she will have traveled?



14. Mr Washington is making a cement table for his backyard. The tabletop will be circular with a diameter of 6 feet and a depth of 6 inches. How much cement will Mr. Washington need to make the top of the table? Use 3.14 for π and round to the nearest cubic foot.

Preparing for Standardized Tests

15. A triangular garden is plotted on grid paper, where each unit is 1 meter. Its sides are segments that are parts of the lines with equations $y = -\frac{5}{4}x + 2$, $2y - 5x = 4$, and $y = -3$. Graph the triangle and find its area.

16. Dylan is flying a kite. He wants to know how high above the ground it is. He knows that he has let out 75 feet of string and that it is flying directly over a nearby fence post. If he is 50 feet from the fence post, how high is the kite? Round to the nearest tenth of a foot.

Measurement

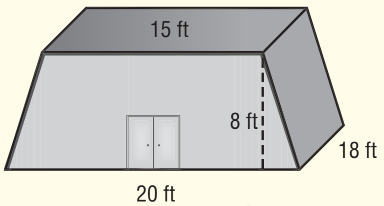
17. The temperature of the Sun can reach 27,000,000°F. The relationship between Fahrenheit F and Celsius C temperatures is given by the equation $F = 1.8C + 32$. Find the temperature of the Sun in degrees Celsius.

18. In 2003, Monaco was the most densely populated country in the world. There were about 32,130 people occupying the country at the rate of 16,477 people per square kilometer. What is the area of Monaco?

19. A box containing laundry soap is a cylinder with a diameter of 10.5 inches and a height of 16 inches. What is the surface area of the box?

20. Light travels at 186,291 miles per second or 299,792 kilometers per second. What is the relationship between miles and kilometers?

21. Home Place Hardware sells storage buildings for your backyard. The front of the building is a trapezoid as shown. The store manager wants to advertise the total volume of the building. Find the volume in cubic feet.



Test-Taking Tip

Questions 15, 16, and 22

Be sure to read the instructions of each problem carefully. Some questions ask for more than one solution, specify how to round answers, or require an explanation.

Data Analysis and Probability

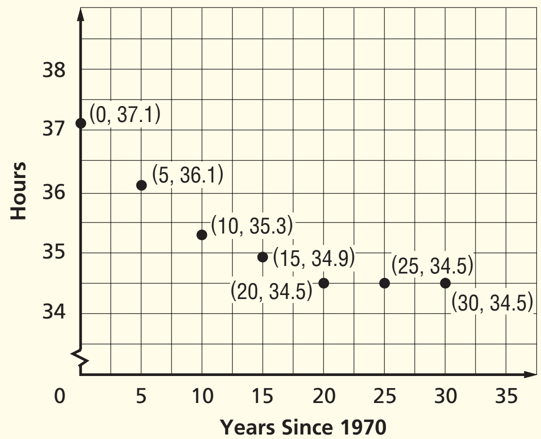
22. The table shows the 2000 populations of the six largest cities in Tennessee. Which measure, mean or median, do you think best represents the data? Explain your answer.

City	Population
Chattanooga	155,404
Clarksville	105,898
Jackson	60,635
Knoxville	173,661
Memphis	648,882
Nashville-Davidson	545,915

Source: International Telecommunication Union

23. The scatter plot shows the number of hours worked per week for U.S. production workers from 1970 through 2000. Let y be the hours worked per week and x be the years since 1970. Write an equation that you think best models the data.

Average Hours Worked per Week for Production Workers



Source: Bureau of Labor Statistics

24. A day camp has 240 participants. Children can sign up for various activities. Suppose 135 children take swimming, 160 take soccer, and 75 take both swimming and soccer. What is the probability that a child selected at random takes swimming or soccer?

25. In how many different ways can seven members of a student government committee sit around a circular table?

26. Illinois residents can choose to buy an environmental license plate to support Illinois parks. Each environmental license plate displays 3 or 4 letters followed by a number 1 thru 99. How many different environmental license plates can be issued?

Preparing for Standardized Tests

Extended-Response Questions

Extended-response questions are often called *open-ended* or *constructed-response questions.* Most extended-response questions have multiple parts. You must answer all parts to receive full credit.

Extended-response questions are similar to short-response questions in that you must show all of your work in solving the problem and a rubric is used to determine whether you receive full, partial, or no credit. The following is a sample rubric for scoring extended-response questions.

Criteria	Credit
Full credit: The answer is correct and a full explanation is provided that shows each step in arriving at the final answer.	2
Partial credit: There are two different ways to receive partial credit. • The answer is correct, but the explanation provided is incomplete or incorrect. • The answer is incorrect, but the explanation and method of solving the problem is correct.	1
No credit: Either an answer is not provided or the answer does not make sense.	0

On some standardized tests, no credit is given for a correct answer if your work is not shown.

Make sure that when the problem says to *Show your work,* you show every aspect of your solution including figures, sketches of graphing calculator screens, or reasoning behind computations.

Preparing for Standardized Tests

Example

Libby throws a ball into the air with a velocity of 64 feet per second. She releases the ball 5 feet above the ground. The height of the ball above the ground t seconds after release is modeled by an equation of the form $h(t) = -16t^2 + v_o t + h_o$ where v_o is the initial velocity in feet per second and h_o is the height at which the ball is released.

a. Write an equation for the flight of the ball. Sketch the graph of the equation.

b. Find the maximum height that the ball reaches and the time that this height is reached.

c. Change only the speed of the release of the ball such that the ball will reach a maximum height greater than 100 feet. Write an equation for the flight of the ball.

Full Credit Solution

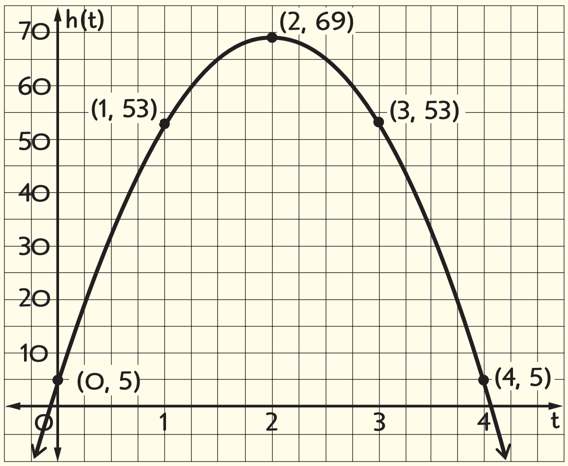
Part a A complete graph includes appropriate scales and labels for the axes, and points that are correctly graphed.

- A complete graph also shows the basic characteristics of the graph. The student should realize that the graph of this equation is a parabola opening downward with a maximum point reached at the vertex.
- The student should choose appropriate points to show the important characteristics of the graph.
- Students should realize that t and x and $h(t)$ and y are interchangeable on a graph on the coordinate plane.

To write the equation for the ball, I substituted $v_o = 64$ and $h_o = 5$ into the equation $h(t) = -16t^2 + v_o t + h_o$, so the equation is $h(t) = -16t^2 + 64t + 5$. To graph the equation, I found the equation of the axis of symmetry and the vertex.

$$x = -\frac{b}{2a}$$
$$= -\frac{64}{2(-16)} \text{ or } 2$$

The equation of the axis of symmetry is $x = 2$, so the x-coordinate of the vertex is 2. You let $t = x$ and then $h(t) = -16t^2 + 64t + 5 = -16(2)^2 + 64(2) + 5$ or 69. The vertex is (2, 69). I found some other points and sketched the graph. I graphed points $(t, h(t))$ as (x, y).



Part b

The maximum height of the ball is reached at the vertex of the parabola. So, the maximum height is 69 feet and the time it takes to reach the maximum height is 2 seconds.

Part c

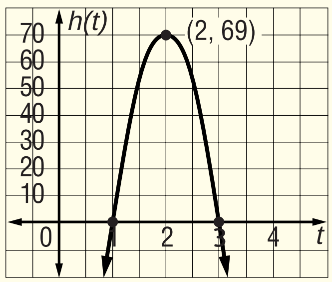
In part c, any equation whose graph has a vertex with *y*-coordinate greater than 100 would be a correct answer earning full credit.

Since I have a graphing calculator, I changed the value of v_o until I found a graph in which the y or $h(t)$ coordinate was greater than 100. The equation I used was $h(t) = -16t^2 + 80t + 5$.

Partial Credit Solution

Part a This sample answer does not earn full credit because it includes no explanation of how the equation was written or the vertex was found.

$h(t) = -16t^2 + 64t + 5$; (2, 69)



CONTENTS

Part b Full credit is given because the vertex is correct and is interpreted correctly.

> The vertex shows the maximum height of the ball. The time it takes to reach the maximum height of 69 feet is 2 seconds.

Part c Partial credit is given for part c since no explanation is given for using this equation. The student did not mention that the vertex would have a *y*-coordinate greater than 100.

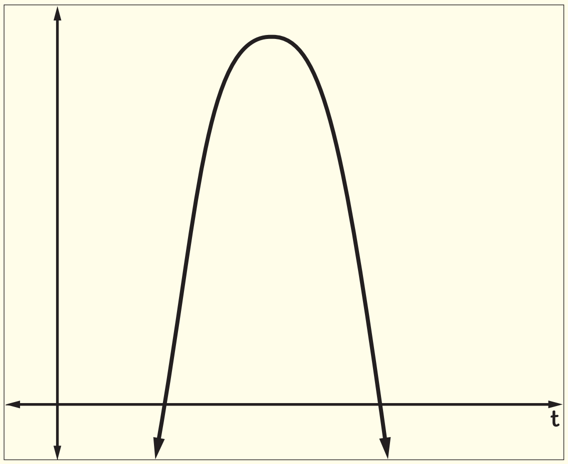
> I will write the equation $h(t) = -16t^2 + 100t + 5$.

This sample answer might have received a score of 2 or 1, depending on the judgment of the scorer. Had the student sketched a more accurate graph and given more complete explanations for Parts a and c, the score would probably have been a 3.

No Credit Solution

Part a No credit is given because the equation is incorrect with no explanation and the sketch of the graph has no labels, making it impossible to determine whether the student understands the relationship between the equation for a parabola and the graph.

> $h(t) = -16t^2 + 5t + 64$



Part b

> It reaches about 10 feet.

Part c

> A good equation for the ball is $h(t) = -16t^2 + 5t + 100$.

In this sample answer, the student does not understand how to substitute the given information into the equation, graph a parabola, or interpret the vertex of a parabola.

Preparing for Standardized Tests

Extended Response Practice

Solve each problem. Show all your work.

Number and Operations

1. Mrs. Ebbrect is assigning identification (ID) numbers to freshman students. She plans to use only the digits 2, 3, 5, 6, 7, and 9. The ID numbers will consist of three digits with no repetitions.

a. How many 3-digit ID numbers can be formed?

b. How many more ID numbers can Mrs. Ebbrect make if she allows repetitions?

c. What type of system could Mrs. Ebbrect use to choose the numbers if there are at least 400 students who need ID numbers?

2. Use these four matrices.

$$A = \begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & -6 & 5 \\ 0 & 1 & -3 \end{bmatrix}$$

$$C = \begin{bmatrix} -7 & 3 \\ -6 & 2 \end{bmatrix} \qquad D = \begin{bmatrix} -3 & 1 \end{bmatrix}$$

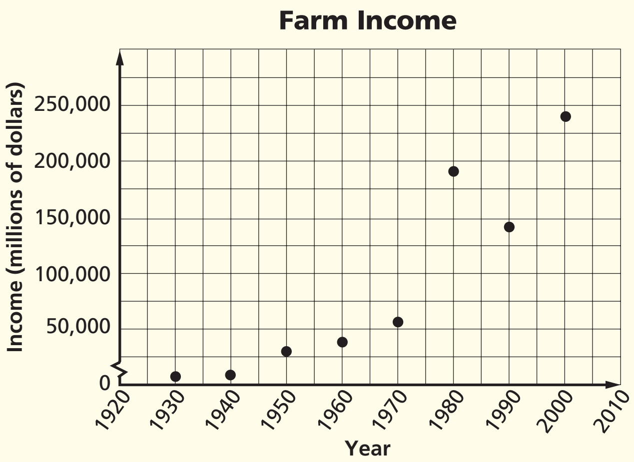
a. Find $A + C$.

b. Compare the dimensions of AB and DB.

c. Compare the matrices BC and CB.

Algebra

3. Roger is using the graph showing the gross cash income for all farms in the U.S. from 1930 through 2000 to make some predictions for the future.



Source: U.S. Department of Agriculture

a. Write an equation in slope-intercept form for the line passing through the point for 1930 and the point for 1970.

b. Write an equation in slope-intercept form for the line passing through the point for 1980 and the point for 2000. Compare the slope of this line to the slope of the line in part a.

c. Which equation, if any, do you think Roger should use to model the data? Explain.

d. Suggest an equation that is not linear for Roger to use.

4. Brad is coaching the bantam age division (8 years old and younger) swim team. On the first day of practice, he has the team swim 4 laps of the 25-meter pool. For each of the next practices, he increases the laps by 3. In other words, the children swim 4 laps the first day, 7 laps the second day, 10 laps the third day, and so on.

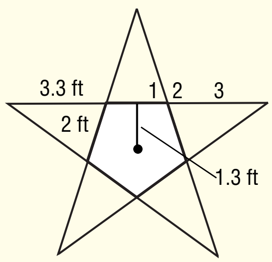
a. Write a formula for the nth term of the sequence of the number of laps each day. Explain how you found the formula.

b. How many laps will the children swim on the 10th day?

c. Brad's goal is to have the children swim at least one mile during practice on the 20th day. If one mile is approximately 1.6 kilometers, will Brad reach his goal?

Geometry

5. Alejandra is planning to use a star shape in a galaxy-themed mural on her wall. The pentagon in the center is regular, and the triangles forming the points are isosceles.

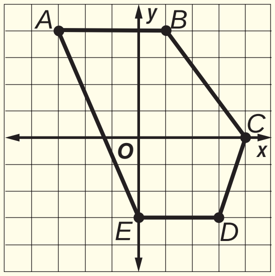


a. Find the measures of ∠1, ∠2, and ∠3. Explain your method.

b. The approximate dimensions of the design are given. The segment of length 1.3 feet is the apothem of the pentagon. Find the approximate area of the design.

c. If Alejandra circumscribes a circle about the star, what is the area of the circle?

6. Kareem is using polygon *ABCDE*, shown on a coordinate plane, as a basis for a computer graphics design. He plans to perform various transformations on the polygon to produce a variety of interesting designs.



a. First, Kareem creates polygon *A′B′C′D′E′* by rotating *ABCDE* counterclockwise about the origin 270°. Graph polygon *A′B′C′D′E′* and describe the relationship between the coordinates of *ABCDE* and *A′B′C′D′E′*.

b. Next, Kareem reflects polygon *A′B′C′D′E′* in the line $y = x$ to produce polygon *A″B″C″D″E″*. Graph *A″B″C″D″E″* and describe the relationship between the coordinates of *A′B′C′D′E′* and *A″B″C″D″E″*.

c. Describe how Kareem could transform polygon *ABCDE* to polygon *A″B″C″D″E″* with only one transformation.

Measurement

7. The speed of a satellite orbiting Earth can be found using the equation $v = \sqrt{\frac{\frac{Gm_E}{r}}{m_E}}$. *G* is the gravitational constant for Earth, m_E is the mass of Earth, and *r* is the radius of the orbit which includes the radius of Earth and the height of the satellite.

a. The radius of Earth is 6.38×10^6 meters. The distance of a particular satellite above Earth is 350 kilometers. What is the value of *r*? (*Hint:* The center of the orbit is the center of Earth.)

b. The gravitational constant for Earth is 6.67×10^{-11} N • m^2/kg^2. The mass of Earth is 5.97×10^{24} kg. Find the speed of the satellite in part a.

c. As a satellite increases in distance from the Earth, what is the effect on the speed of the orbit? Explain your reasoning.

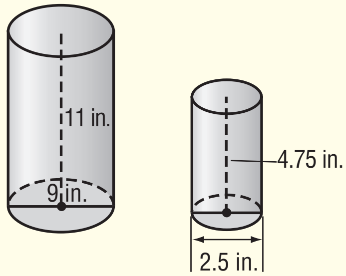
Test-Taking Tip

A B C D

Question 6

When questions require graphing, make sure your graph is accurate to receive full credit for your correct solution.

8. A cylindrical cooler has a diameter of 9 inches and a height of 11 inches. Scott plans to use it for soda cans that have a diameter of 2.5 inches and a height of 4.75 inches.



a. Scott plans to place two layers consisting of 9 cans each into the cooler. What is the volume of the space that will not be filled with cans?

b. Find the ratio of the volume of the cooler to the volume of the cans in part b.

Data Analysis and Probability

9. The table shows the total world population from 1950 through 2000.

Year	Population	Year	Population
1950	2,566,000,053	1980	4,453,831,714
1960	3,039,451,023	1990	5,278,639,789
1970	3,706,618,163	2000	6,082,966,429

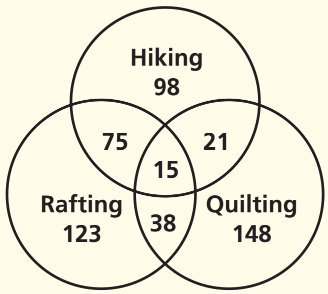
a. Between which two decades was the percent increase in population the greatest?

b. Make a scatter plot of the data.

c. Find a function that models the data.

d. Predict the world population for 2030.

10. Each year, a university sponsors a conference for women. The Venn diagram shows the number of participants in three activities for the 680 women that attended. Suppose women who attended are selected at random for a survey.



a. What is the probability that a woman selected participated in hiking or sculpting?

b. Describe a set of women such that the probability of their being selected is about 0.39.

Preparing for Standardized Tests